# Supply Concepts

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ECON 306

## Firm's Constrained Optimization

- The **Firms (constrained optimization) problem** is:
  1. **Choose:** <inputs, output>
  2. **In order to maximize:** <profits>
  3. **Subject to**: <technology>
- We break up the firm's problem into two problems:
- The firm's **cost-minimization problem**:
  1. **Choose:** <inputs>
  2. **In order to minimize:** <total cost>
  3. **Subject to**: <producing optimal output>
- The firm's **profit-maximization problem**:
  1. **Choose:** <output>
  2. **In order to maximize:** <profit>

### Production & Firms

- Firms organize production by buying or renting inputs ("factors of production") and transforming them into outputs according to their **technology** or **production function**

  $$q = f(k, l)$$

  where $q$ = amount of output, $k$ = amount of capital, and $l$ = amount of labor
- Two time-frames of production:
  - **Short-run**: at least one factor of production is fixed (e.g. $\bar{k}$)
    - We can characterize the short-run production function by plugging in the amount of our fixed factor, e.g.

      $$\begin{aligned} q(l, k) &= lk \\ \bar{k} &= 10 \\ q(l, \bar{k}) &= 10l \end{aligned}$$

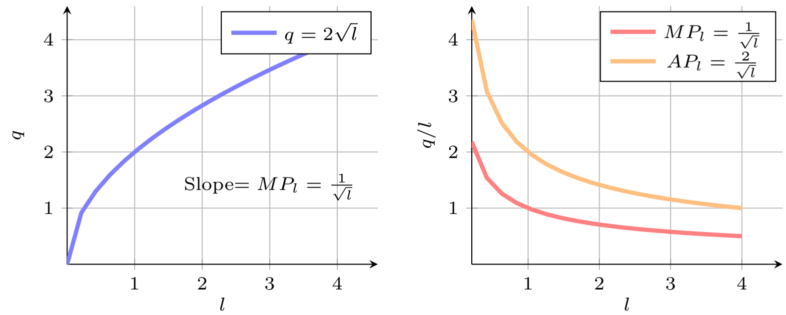



Table 1: Short-run production function with diminishing returns

- * The **marginal product** of an input measures how output changes as one input is added (holding the other(s) constant):

$$MP_l = \frac{\Delta q}{\Delta l}$$
$$MP_k = \frac{\Delta q}{\Delta k}$$

  - · Inputs are often assumed to have **diminishing returns**: $MP$ is declining ($q$ is increasing at a decreasing rate with respect to each input)

- * The **average product** of an input measures output per unit of input

$$AP_l = \frac{q}{l}$$
$$AP_k = \frac{q}{k}$$

- – **Long-run**: all factors are variable

## Isocost Lines

- **Isocost line**: the combinations of inputs that are the same total cost

$$wl + rk = C$$

$w$ = price of labor, $r$ = price of capital

- – To graph, solve for $k$:

$$k = \frac{C}{r} - \frac{w}{r}l$$

* Vertical intercept: $\dfrac{C}{r}$
* Horizontal intercept: $\dfrac{C}{w}$
* Slope: $-\dfrac{w}{r}$

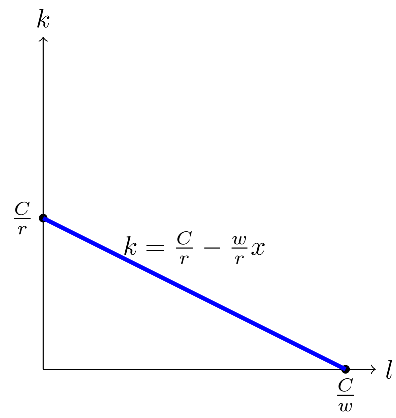
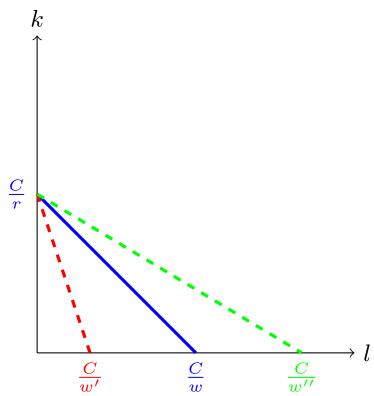



Figure 1: The Isocost Line

- All points on the line are same total cost
  - All points beneath line are lower total cost
  - All points above the line are higher total cost
- Change in an input's market price: rotates isocost line
  - New intercept for input that changed in price
  - New slope
- Slope of isocost line measures the *market* exchange rate between $l$ and $k$ (their relative prices)

$\Delta w$ ($\uparrow$ in red, $\downarrow$ in green)




$\Delta r$ ($\uparrow$ in red, $\downarrow$ in green)

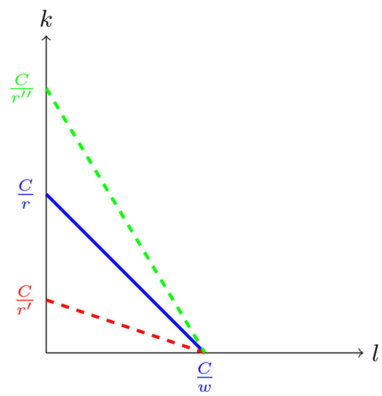



Table 2: How the isocost line changes with input prices

## Isoquant Curves

- **Isoquant curves** link all combinations of inputs that produce the same output

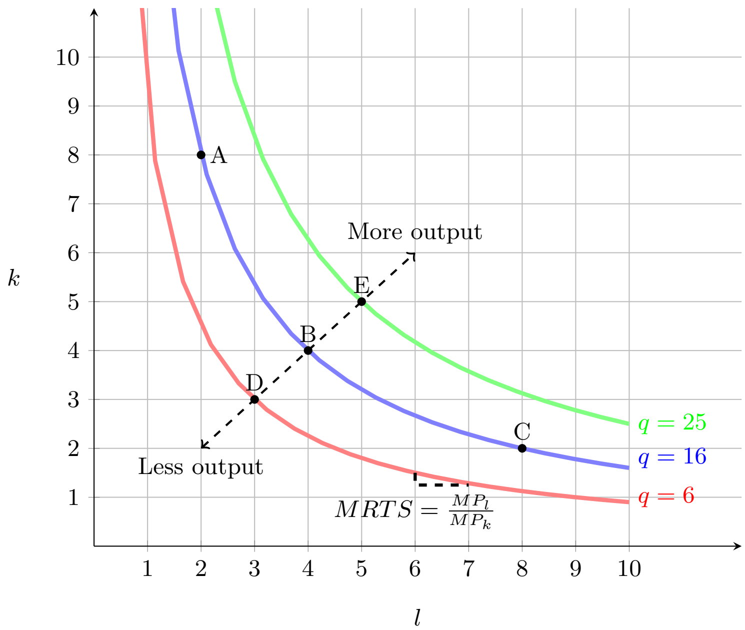



Figure 2: Isoquant curves: $E > A = B = C > D$

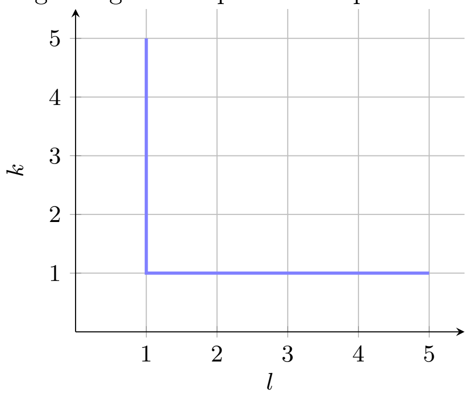
  - **Marginal rate of technical substitution (MRTS)**: firm's exchange rate between $l$ and $k$
    - $MRTS =$ the slope of the isoquant curve
    - Literally: the amount of $k$ given up to obtain 1 more $k$ produce same output
  - Marginal products are related to MRTS:

$$MRTS = \frac{MP_l}{MP_k}$$

– Shape & slopes (MRTS) of isoquant curves:

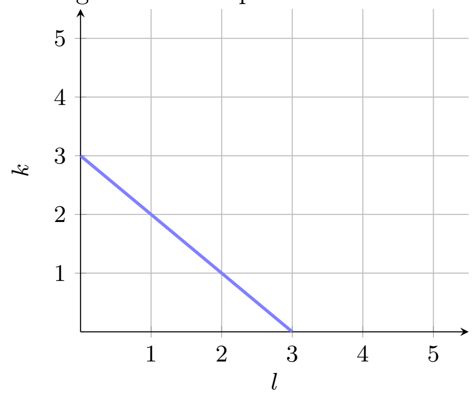
  * Bent vs. straight $\implies$ complementarity vs. substitutability between $l$ and $k$

Right-angle $\implies$ perfect complements




Always produce at same rate of combination

Straight line $\implies$ perfect substitutes




Always substitute at same rate

## Solving the Firm's Cost-Minimization Problem

- Firm chooses combination of $l$ and $k$ to minimize total cost while producing the optimal amount of output
  - Expressed mathematically:

$$\min_{l,k} wl + rk$$

$$\text{s. t. } q^* = f(k,l)$$

  - Graphically: optimum is the point of tangency between the lowest isocost line tangent to the (optimal) isoquant

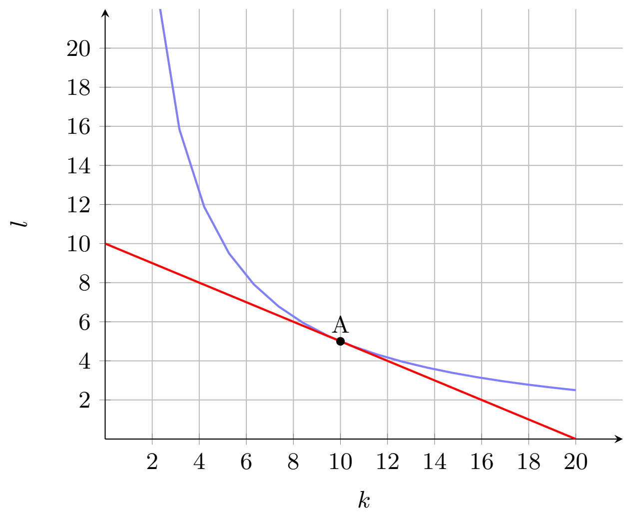



Figure 3: The firm's optimum at point $A$: isoquant curve is tangent to isocost line

  - At the tangency point ($A$), all of the following are true:

| | |
|---|---|
| $\lvert\text{Slope of I.Q. Curve}\rvert = \lvert\text{Slope of I.C. Line}\rvert$ | Slopes are equal |
| $MRTS = \frac{w}{r}$ | Definition of each slope |
| $\frac{MP_l}{MP_k} = \frac{w}{r}$ | Firm's exchange rate same as market exchange rate |
| $\frac{MP_l}{w} = \frac{MP_k}{r}$ | Marginal product per \$1 is the same between $l$ and $k$ |

- **Equimarginal principle**: output is optimized when firm can lower costs no more output by spending \$1 more/less on either $l$ or $k$
  - Firm is indifferent between using more $l$ or using more $k$: has no reason to change input decisions!
  - If marginal product per dollar were greater for (e.g.) $l$ than for $k$, could buy more $l$ and lower costs!

- **Returns to Scale**: technological relationship between scaling all inputs at the same rate and the scale of output
  - Constant returns to scale: output scales at the same rate as scaling all inputs
    - e.g. doubling all inputs doubles output

- Increasing returns to scale: output scales at a faster rate than scaling all inputs
  - e.g. doubling all inputs more-than-doubles output
- Decreasing returns to scale: output scales at a slower rate than scaling all inputs
  - e.g. doubling all inputs less-than-doubles output

# Supply in Competitive Markets

## Costs

- Economic vs. accounting concepts:
  - Accounting costs: monetary costs
  - Economic (opportunity) costs: value of next best opportunity given up
  - Accounting profit: Total revenue minus accounting costs
  - Economic profit: Total revenue minus accounting & economic costs
  - Accounting point of view: are you taking in more cash than you are spending
  - Economic point of view: are you really making the *best* use of your resources with your current project (i.e. is there a higher-value use)?
    * Implications for society: consumers really *best* off with you using scarce resources (with other uses) to produce your current product?
- Total cost function $C(q)$ relates total quantity of output $q$ (using optimal combinations of $l$ and $k$) to the total cost of production $C$

$$C(q) = FC + VC(q)$$

  - Fixed Costs $FC$: costs that do not vary with output
  - Average Fixed Costs $AFC(q)$: fixed costs per unit

$$AFC(q) = \frac{FC}{q}$$

  - Variable Costs $VC(q)$: costs that vary with output
  - Average Variable Cost $AVC(q)$: variable cost per unit of output

$$AVC(q) = \frac{VC}{q}$$

  - Average (Total) Cost $AC(q)$: (total) cost per unit of output

$$AC(q) = \frac{TC}{q}$$
$$AC(q) = AFC(q) + AVC(q)$$

  - Marginal Cost ($MC(q)$): how cost changes with one unit of output

$$MC(q) = \frac{\Delta C(q)}{\Delta q}$$

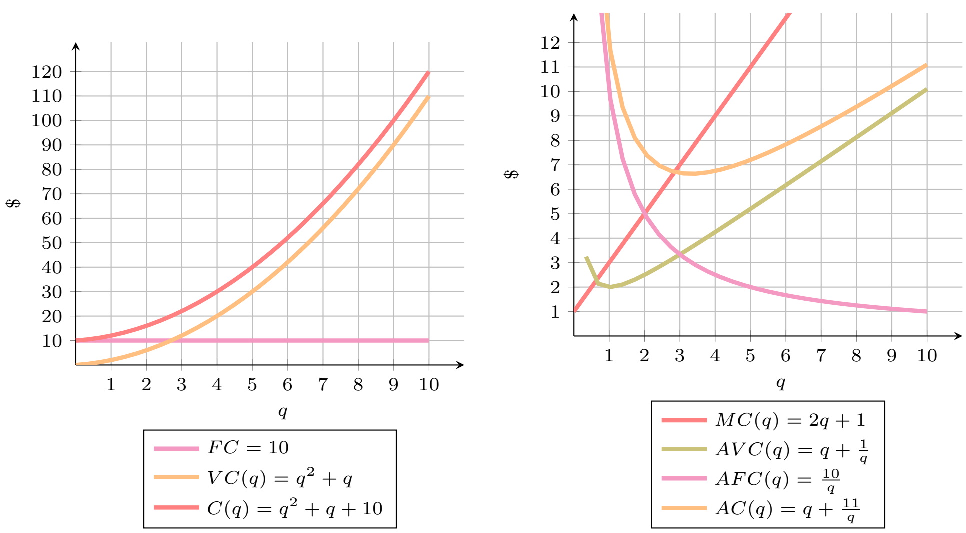



Table 3: Total costs (left) and per-unit costs (right)

- General relationship between average and marginal:
  - When $MC(q) > AC(q)$, $\uparrow AC(q)$
  - When $MC(q) < AC(q)$, $\downarrow AC(q)$
  - When $MC(q) = AC(q)$, $AC(q)$ is minimized
  - Same relationship between $MC$ and $AVC$

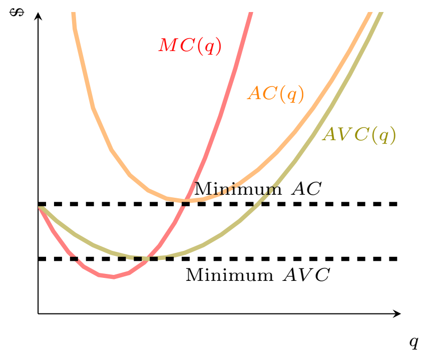



Figure 4: The relationship between average and marginal

∗ In the long run, firms can change all factors of production (e.g. can choose $k$)

- Separate short run average cost curves for each hypothetical amount of $k$
- In long run, firm chooses $k$ (and associated SRAC curve) to minimize cost at desired output level
- Long run average cost curve "envelopes" the lowest parts of all SRAC curves

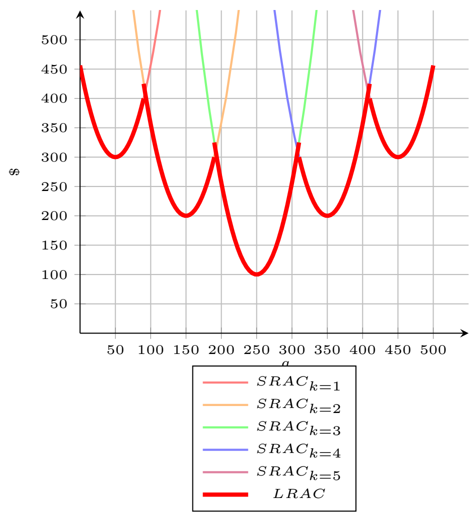
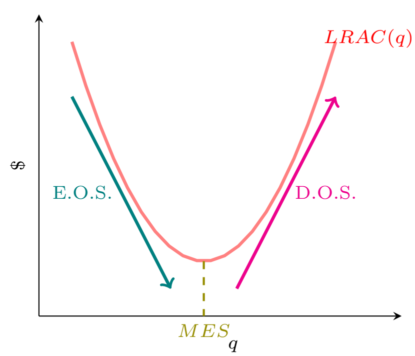



Figure 5: The relationship between short and long run average cost curves

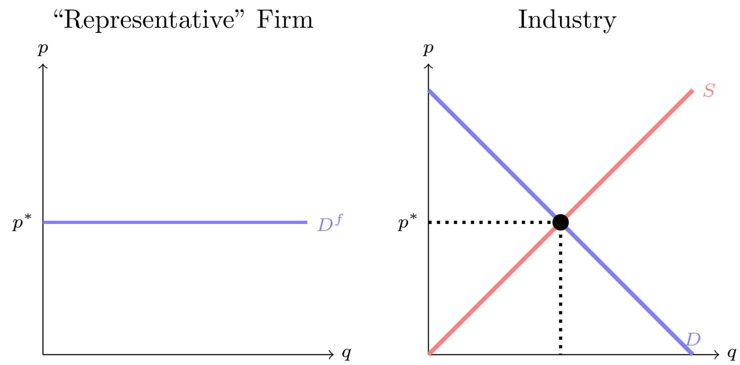
∗ **Economies of scale**: the economic relationship between how average cost scales with output

- Economies of scale: average costs fall with output
- Diseconomies of scale: average costs rise with output
- Constant economies of scale: costs do not vary with output
- Minimum efficient scale (MES): $q$ with lowest $AC(q)$



## Revenues

– *Competitive* price-taking firm's demand is *perfectly elastic* at the market-determined price




– Total revenue

$$R(q) = pq$$

∗ Average Revenue: revenue per unit (aka price)

$$AR(q) = p$$

∗ Marginal Revenue: how revenues change with one more output

$$MR(q) = \frac{\Delta R(q)}{\Delta q}$$

· For a *price-taking* firm in a *competitive* market, Demand= $AR(q) = MR(q) = p$

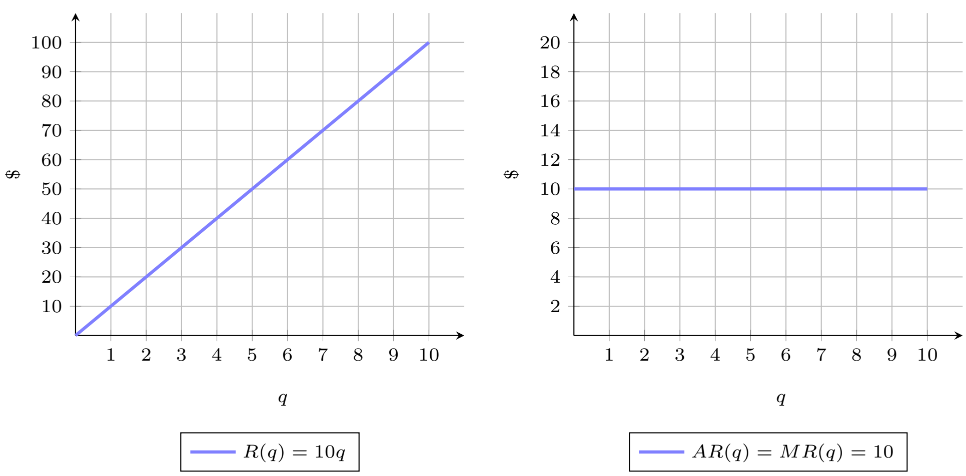
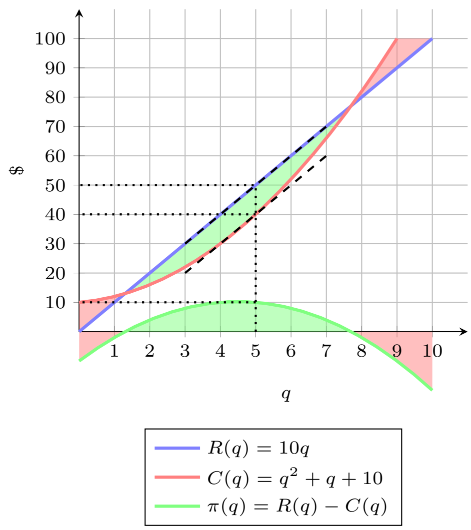



Table 4: Firm's total (left) and per-unit (right) revenues

## Profits

- A **competitive market**:
  - Firms' products are perfect substitutes
  - Firms are price-takers, none can affect the market price
  - Market entry and exit is costless
- Firm chooses profit maximizing quantity $q^*$:

$$\pi_{max} \text{ at } q^* \text{ where } MR(q) = MC(q)$$



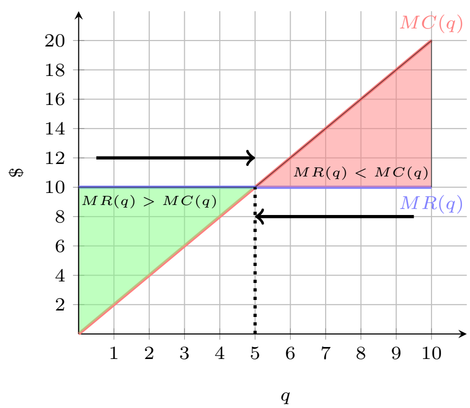
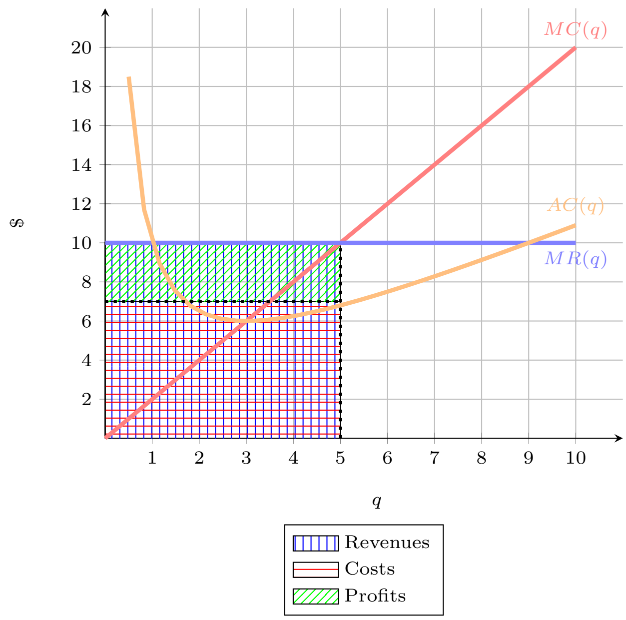




Table 5: Finding maximum profits (totals on left, per unit on right)

- Profit is revenues minus costs:

$$\pi = R(q) - C(q)$$
$$\pi = q[p - AC(q)]$$




- Firm breaks even where $p = AC(q)$
  - Firm's break even price is the minimum of $AC(q)$ curve (where $AC(q) = MC(q)$)
- Firm earns losses where $p < AC(q)$
  - **Short run**: firm stays in market
    - Firm continues to produce (at a loss) if

$$p \geq AVC$$

    - Firm **shuts down** and produces $q^* = 0$ if

$$p < AVC$$

    - Firm's shut down price is the minimum of $AVC(q)$ curve (where $AVC(q) = MC(q)$)
  - **Long run**: firm exits market

- Firm's Supply:

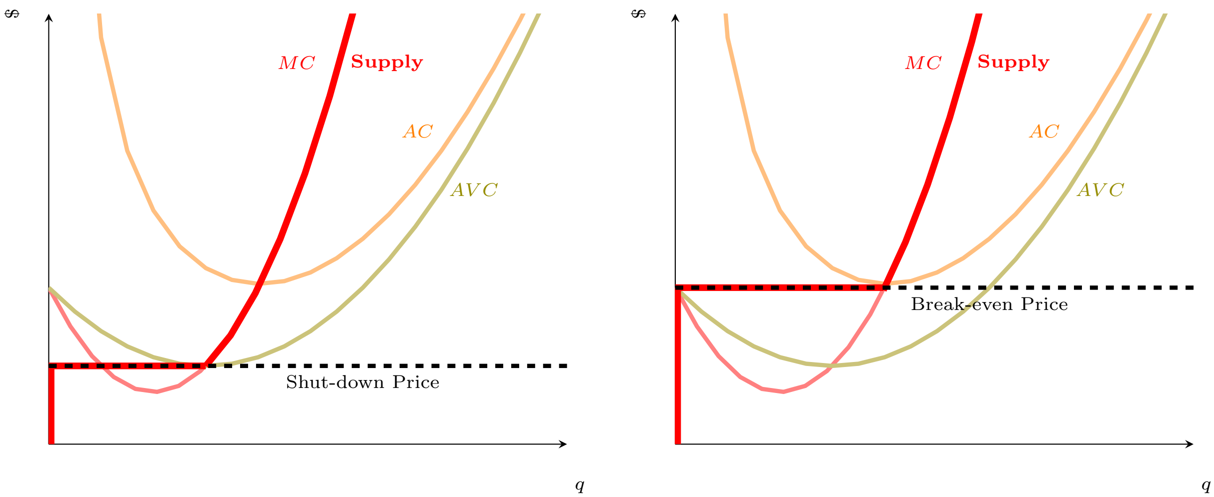
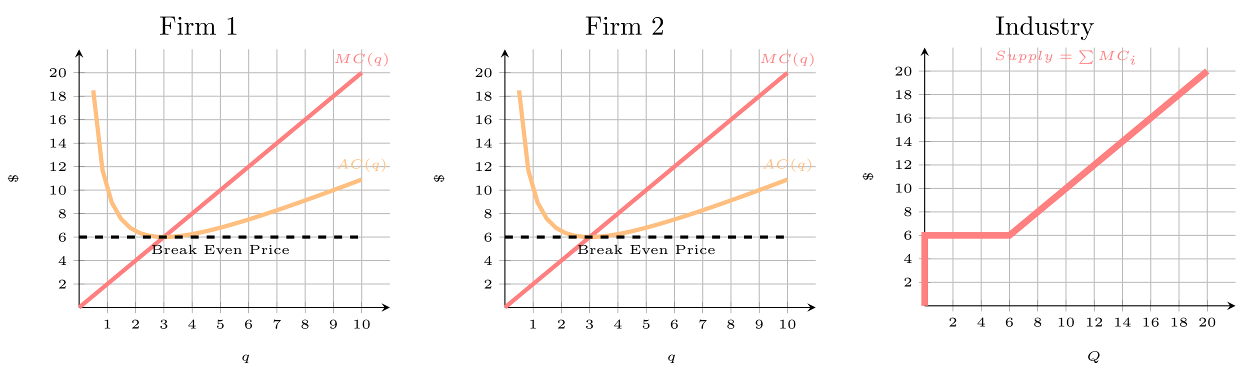



Table 6: Firm's Supply in Short Run (left) and Long Run (right)

$$\text{Firm's Short Run Inverse Supply} = \begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

$$\text{Firm's Long Run Inverse Supply} = \begin{cases} p = MC(q) & \text{if } p \geq AC \\ q = 0 & \text{If } p < AC \end{cases}$$

- Industry equilibrium:
  - If firms earn $\pi > 0$ in short run: firms enter over long run
  - If firms earn $\pi < 0$ in short run: firms exit over long run
  - **Long run equilibrium:** $\pi = 0$ **at** $p = AC(q) = MC(q)$ **for all firms!**
- Industry supply curve is sum of all firms' marginal cost curves above $AVC_{min}$




- Firms may have different cost structures due to **economic rents** – returns above opportunity cost needed to bring firm online
  - A scarce factor of production (e.g. talent, location, intellectual property, political favors, etc)
  - Lowers costs for firm relative to other firms

- Other firms willing to bid up price of scarce rent-generating factor (to earn advantage)
- Prices of rent-generating factors get bid up until firm profits fall to zero!
- Owner of scarce factor earns higher income due to economic rents