1.3 — Preferences

ECON 306 • Microeconomic Analysis • Spring 2023

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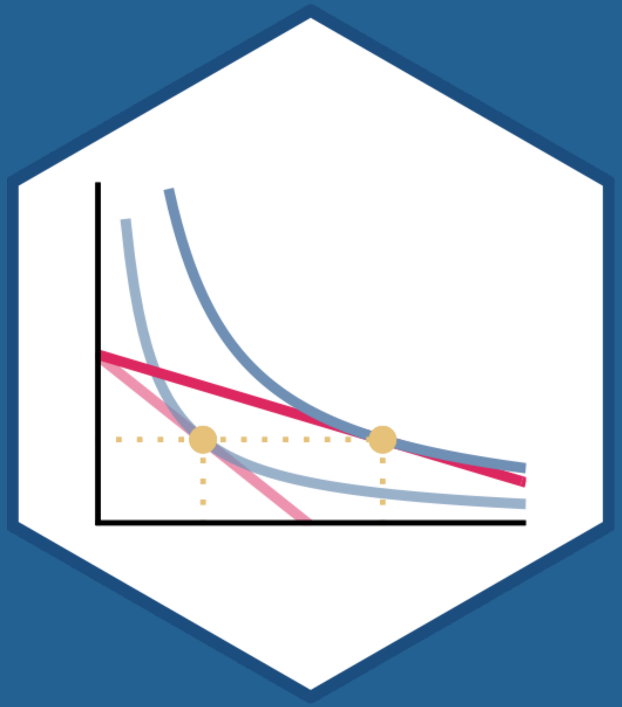
[ryansafner/microS23](https://github.com/ryansafner/microS23)

microS23.classes.ryansafner.com



Outline

Preferences

Preferences I

- Which bundles are **preferred** over others?

Example: Between two bundles of (x, y):

$$a = (4, 12) \text{ or } b = (6, 12)$$

Preferences II

- We will allow **three possible answers**:

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1. $a \succ b$: (Strictly) prefer a over b

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2. $a \prec b$: (Strictly) prefer b over a

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1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a
3. $a \sim b$: Indifferent between a and b

Preferences II

- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a
3. $a \sim b$: Indifferent between a and b

- ***Preferences*** **are a list of all such comparisons between all bundles**

See appendix in today's class page for more.

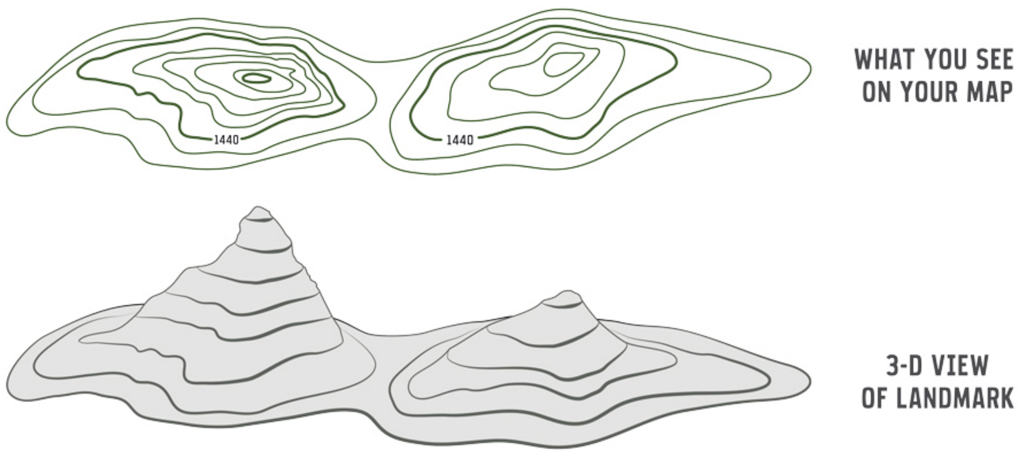
Indifference Curves

Mapping Preferences Graphically I

- For each bundle, we now have 3 pieces of information:
  - amount of x
  - amount of y
  - preference compared to other bundles
- How to represent this information graphically?

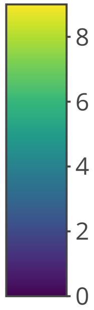
Mapping Preferences Graphically II

- Cartographers have the answer for us
- On a map, **contour lines** link areas of **equal height**
- We will use “indifference curves” to link bundles of **equal preference**

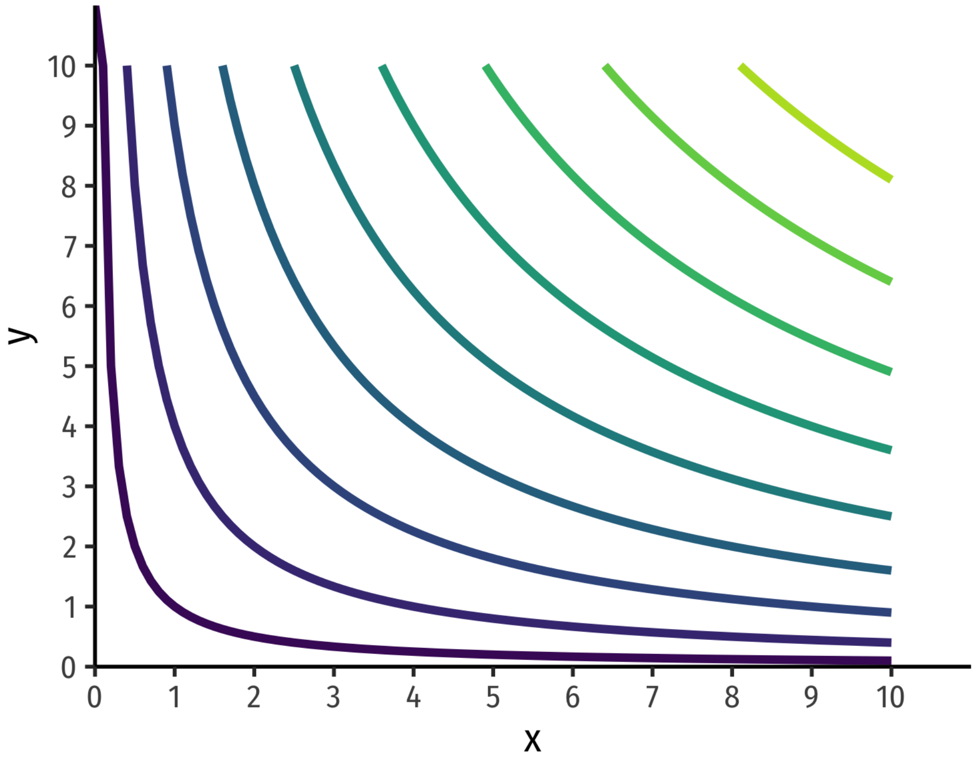


Mapping Preferences Graphically III

3-D “Mount Utility”

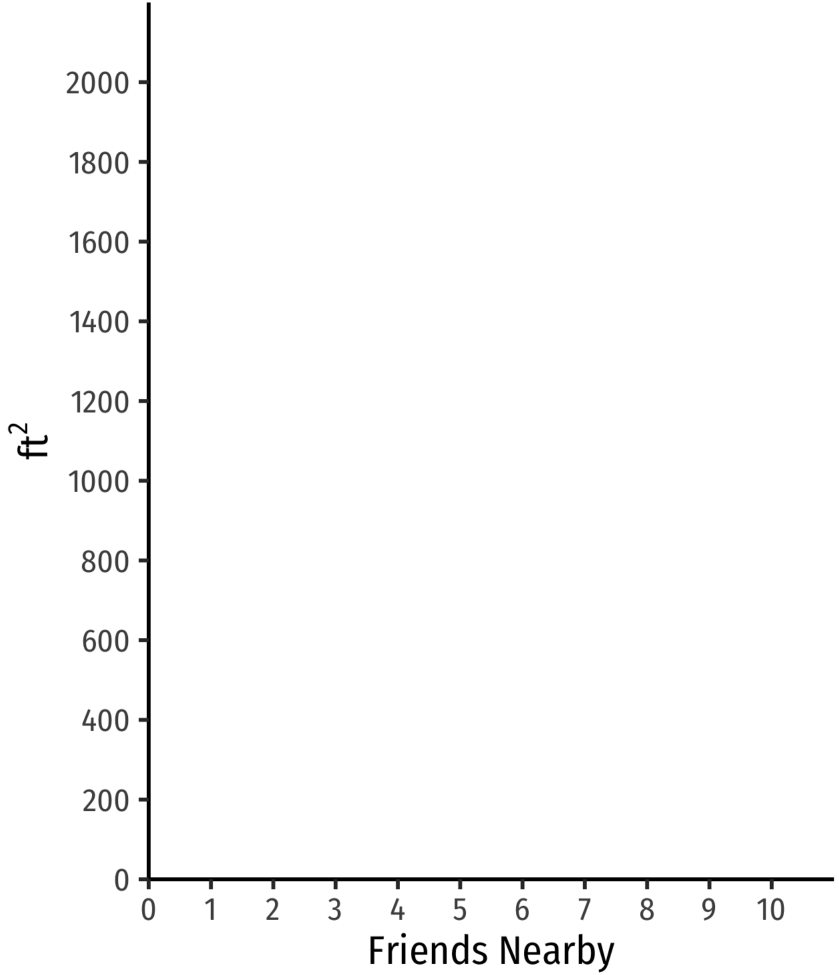


2-D Indifference Curve Contours



Indifference Curves: Example

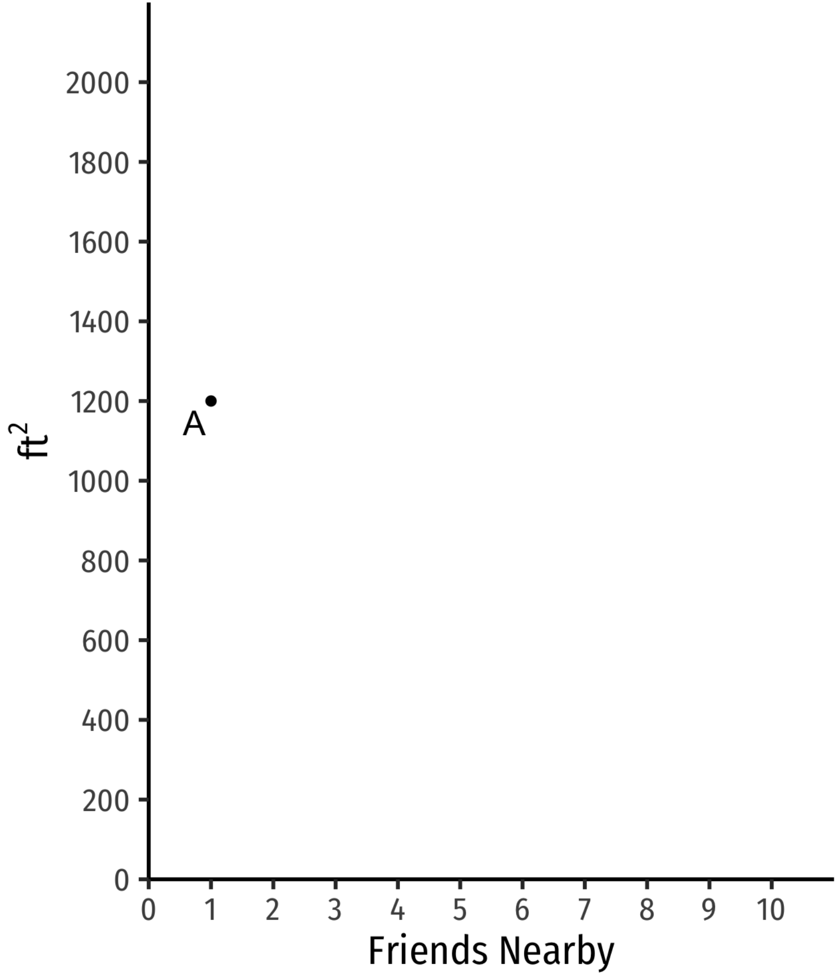
Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.



Indifference Curves: Example

Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

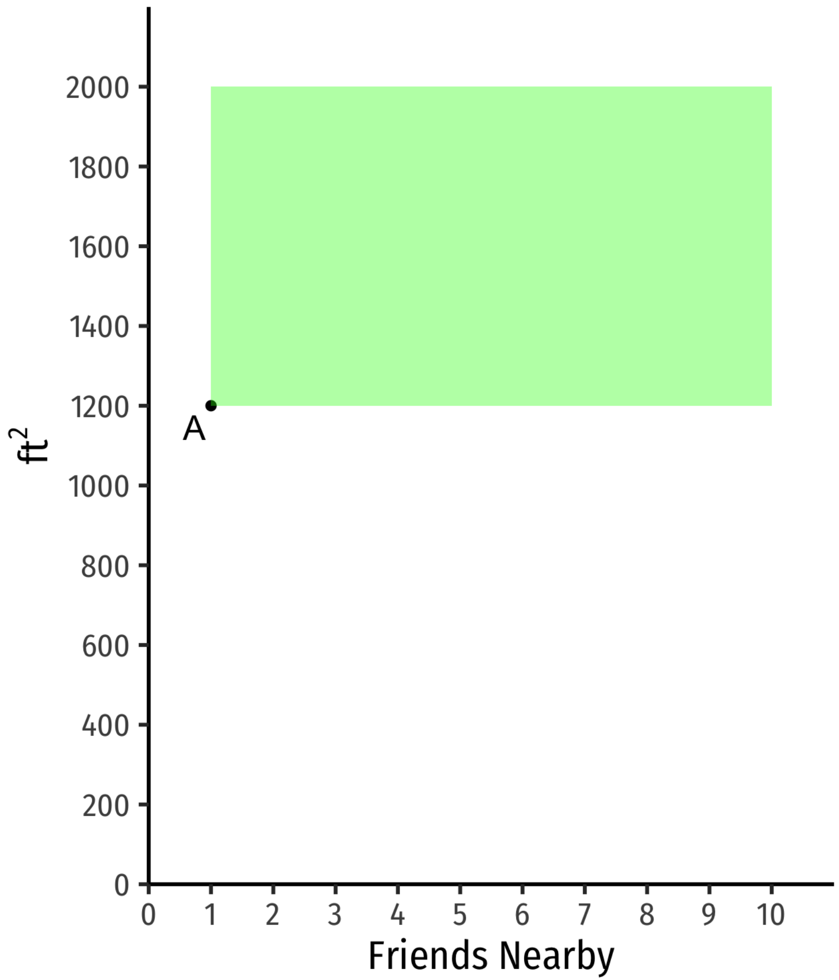
- Apt. *A* has 1 friend nearby and is 1,200 ft^2



Indifference Curves: Example

Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

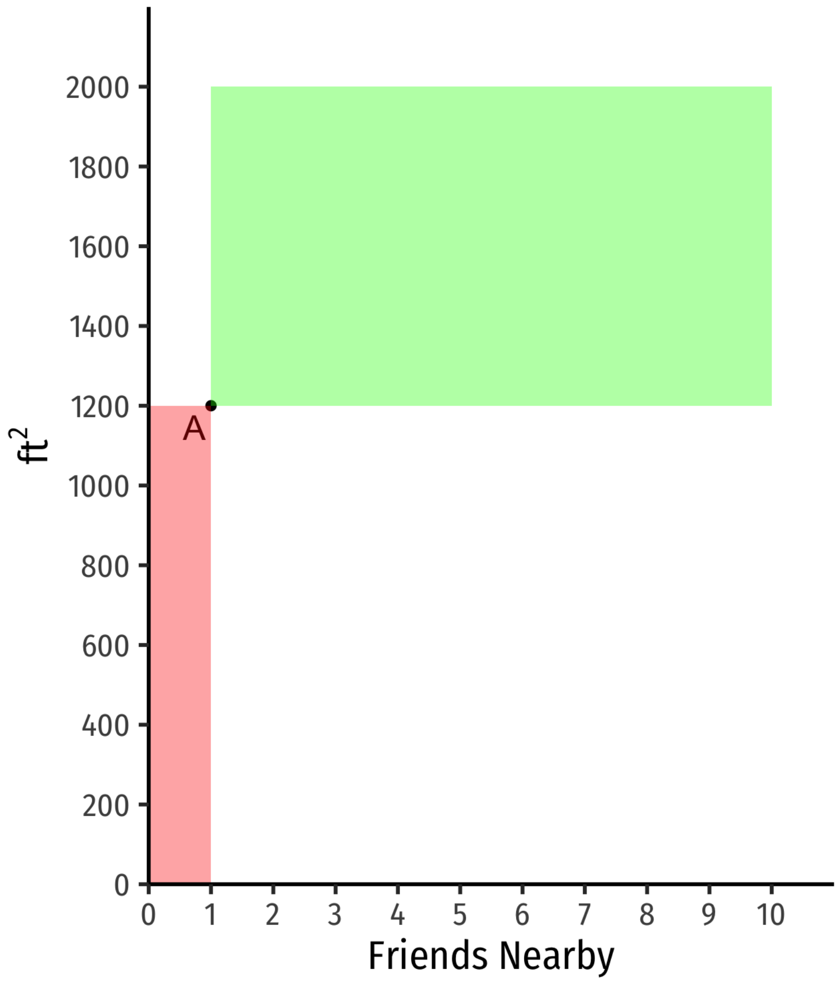
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
  - Apts that are larger and/or have more friends $\succ A$



Indifference Curves: Example

Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

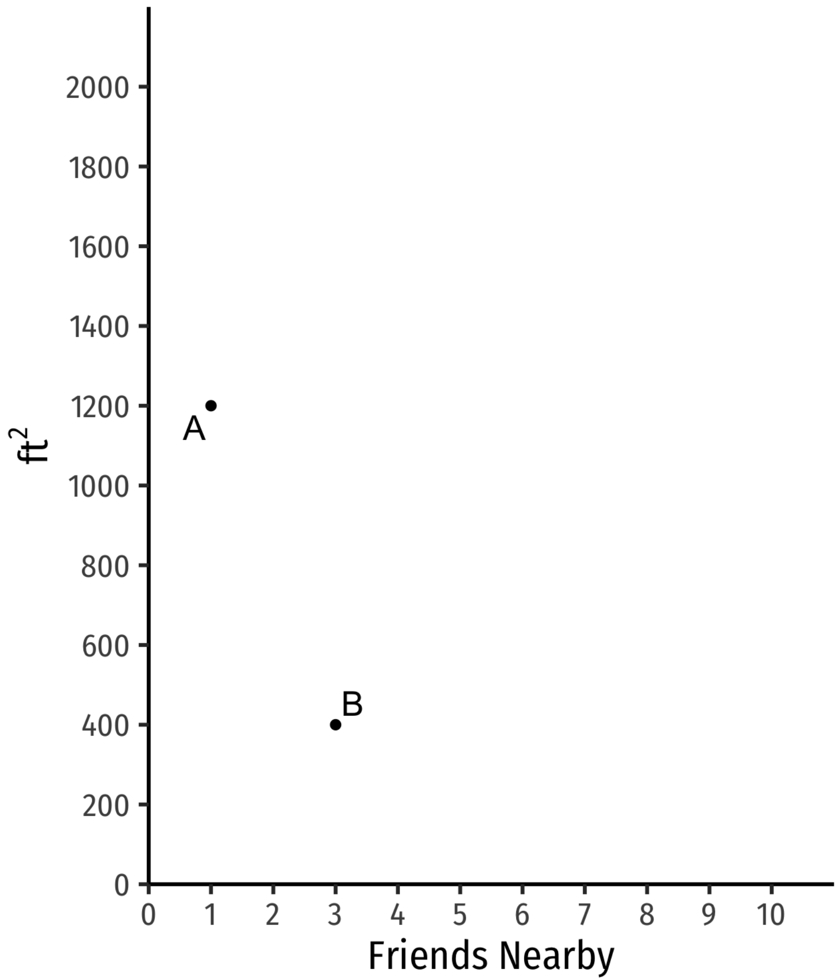
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
  - Apts that are larger and/or have more friends $\succ A$
  - Apts that are smaller and/or have fewer friends $\prec A$



Indifference Curves: Example

Example:

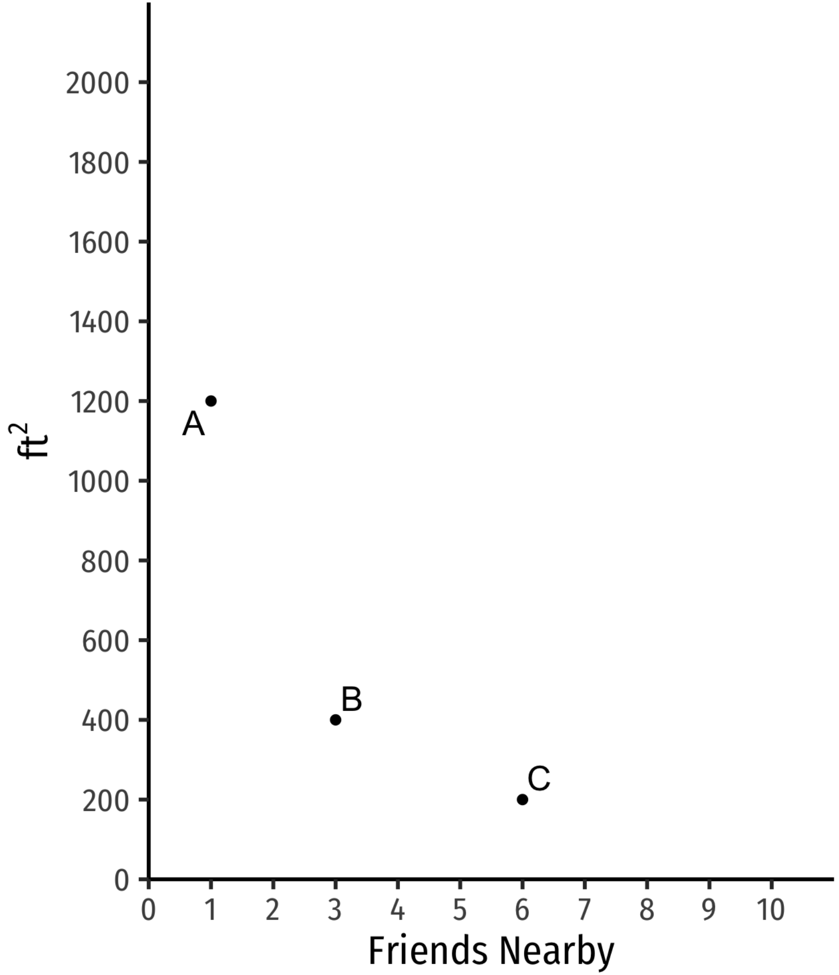
- Apt. A has 1 friend nearby and is 1,200 ft^2
- B has *more* friends but *less* ft^2



Indifference Curves: Example

Example:

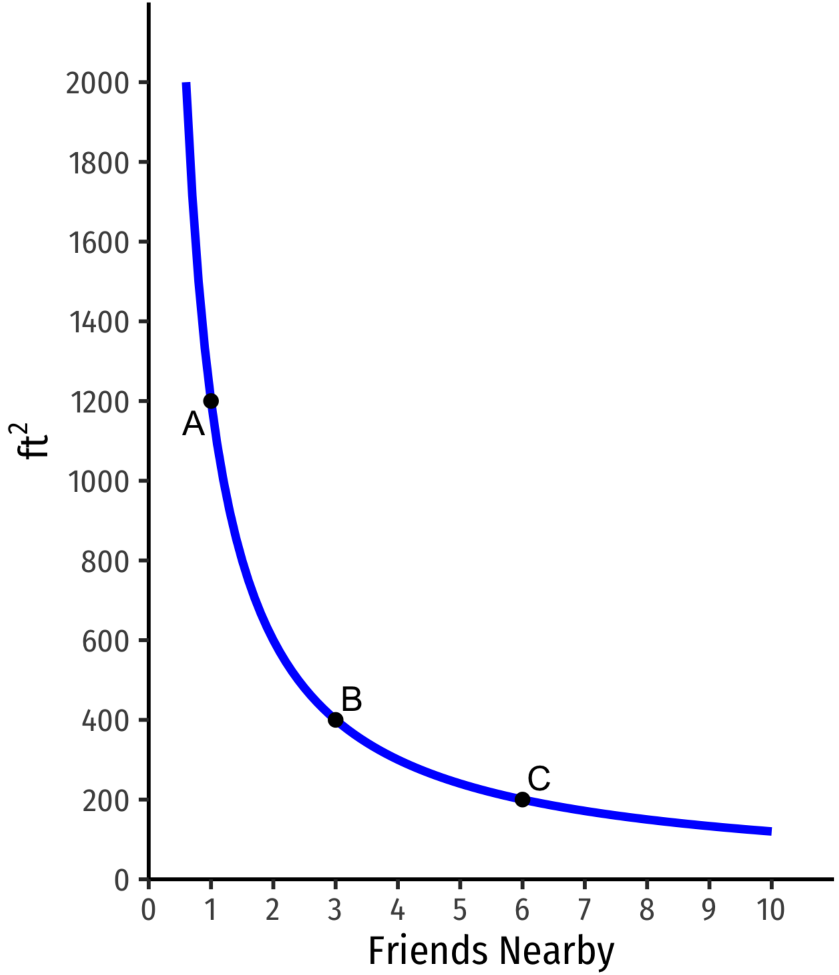
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- *B* has *more* friends but *less* ft^2
- *C* has *still more* friends but *less* ft^2



Indifference Curves: Example

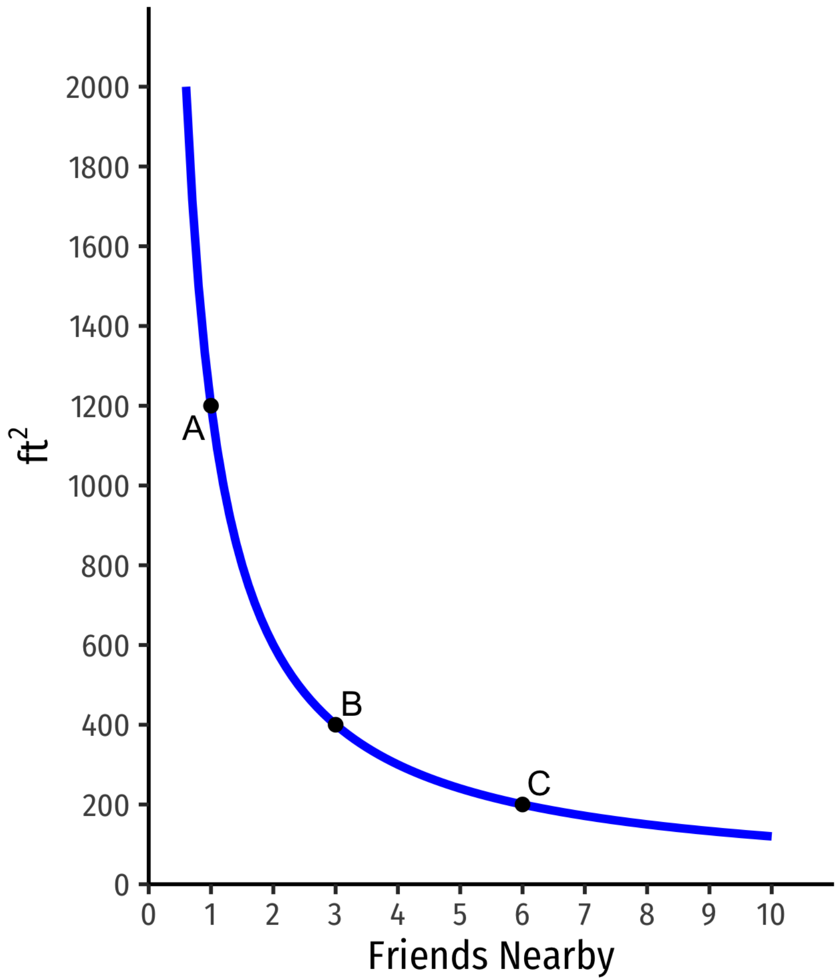
Example:

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- *B* has *more* friends but *less* ft^2
- *C* has *still more* friends but *less* ft^2
- $A \sim B \sim C$: on same **indifference curve**



Indifference Curves: Example

- **Indifferent** between all apartments on the **same** curve

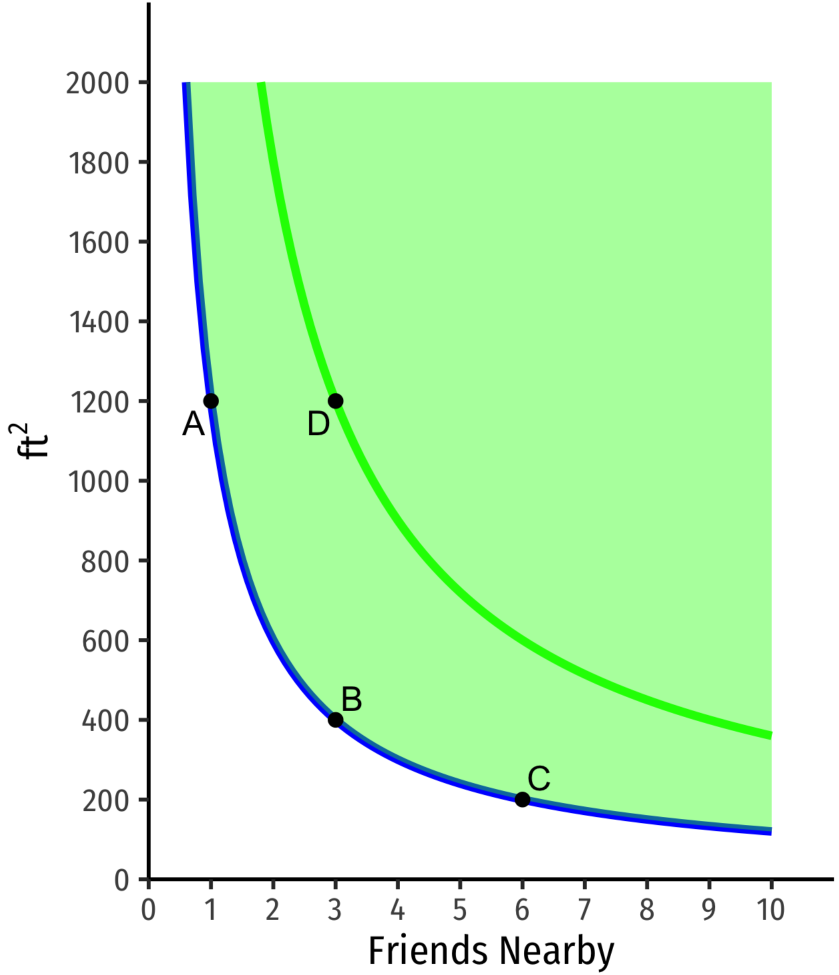


Indifference Curves: Example

- **Indifferent** between all apartments on the **same** curve

- Apts **above** curve are **preferred over** apts on curve

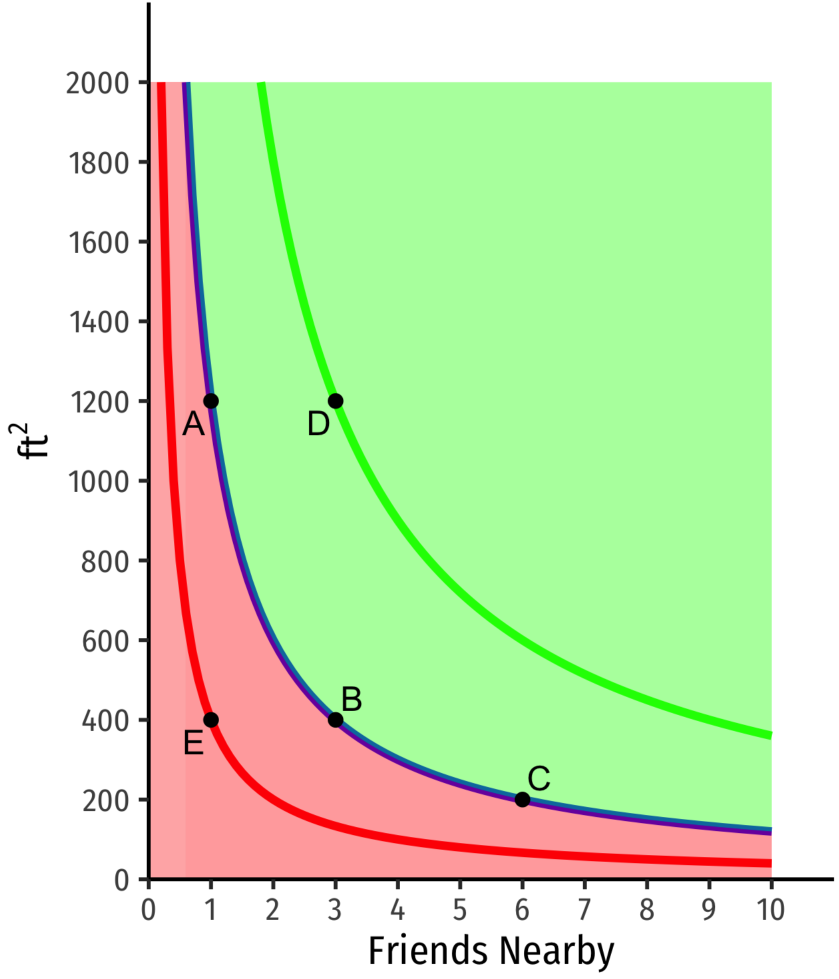
  - $D \succ A \sim B \sim C$
  - On a **higher curve**



Indifference Curves: Example

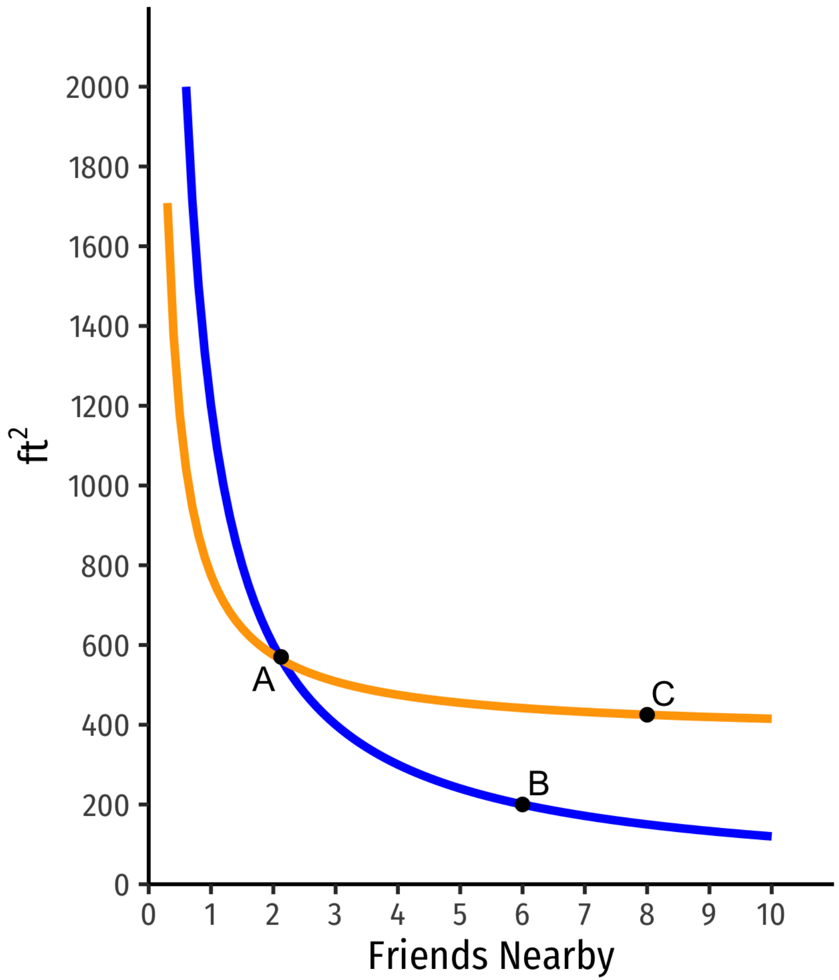
- **Indifferent** between all apartments on the **same** curve

- Apts **above** curve are **preferred over** apts on curve
  - $D \succ A \sim B \sim C$
  - On a **higher curve**
- Apts **below** curve are **less preferred** than apts on curve
  - $E \prec A \sim B \sim C$
  - On a **lower curve**



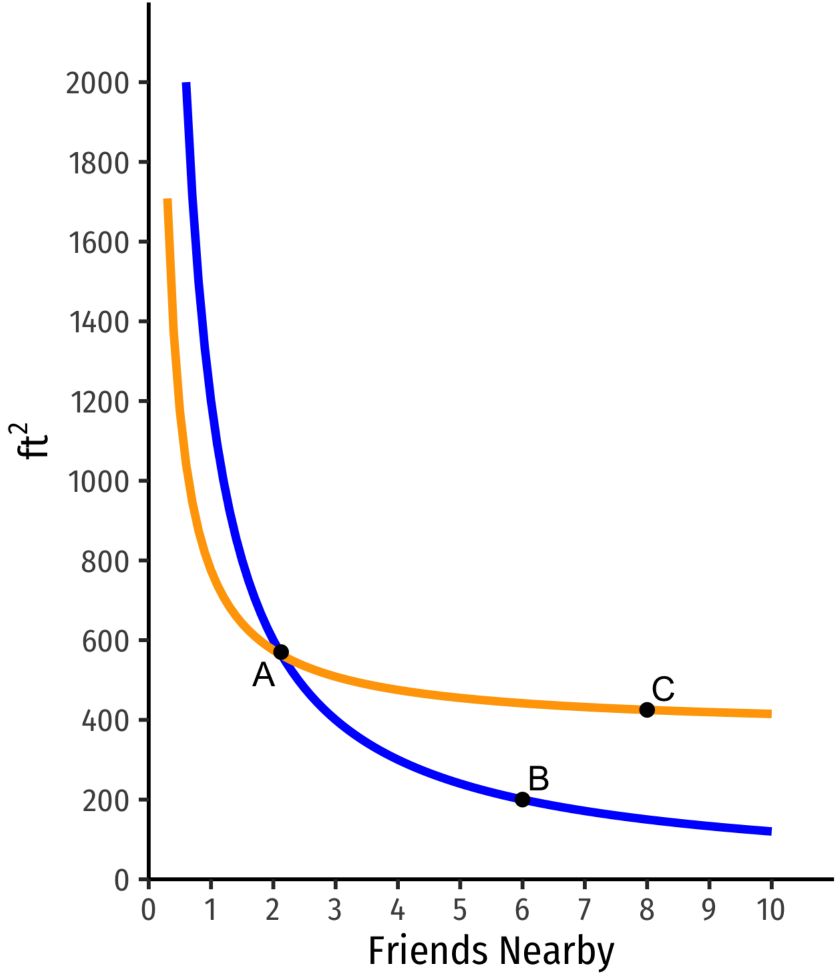
Curves Never Cross!

- **Indifference curves can never cross**: preferences are **transitive**
  - If I prefer $A \succ B$, and $B \succ C$, I must prefer $A \succ C$



Curves Never Cross!

- **Indifference curves can never cross**: preferences are **transitive**
    - If I prefer $A \succ B$, and $B \succ C$, I must prefer $A \succ C$
- Suppose two curves crossed:
    - $A \sim B$
    - $B \sim C$
    - But $C \succ B$!
    - Doesn't make sense (not transitive)!



Marginal Rate of Substitution

Marginal Rate of Substitution I

- If I find another apt with *1 fewer friend* nearby, how many *more* ft^2 would you need to keep you *satisfied?*

Marginal Rate of Substitution I

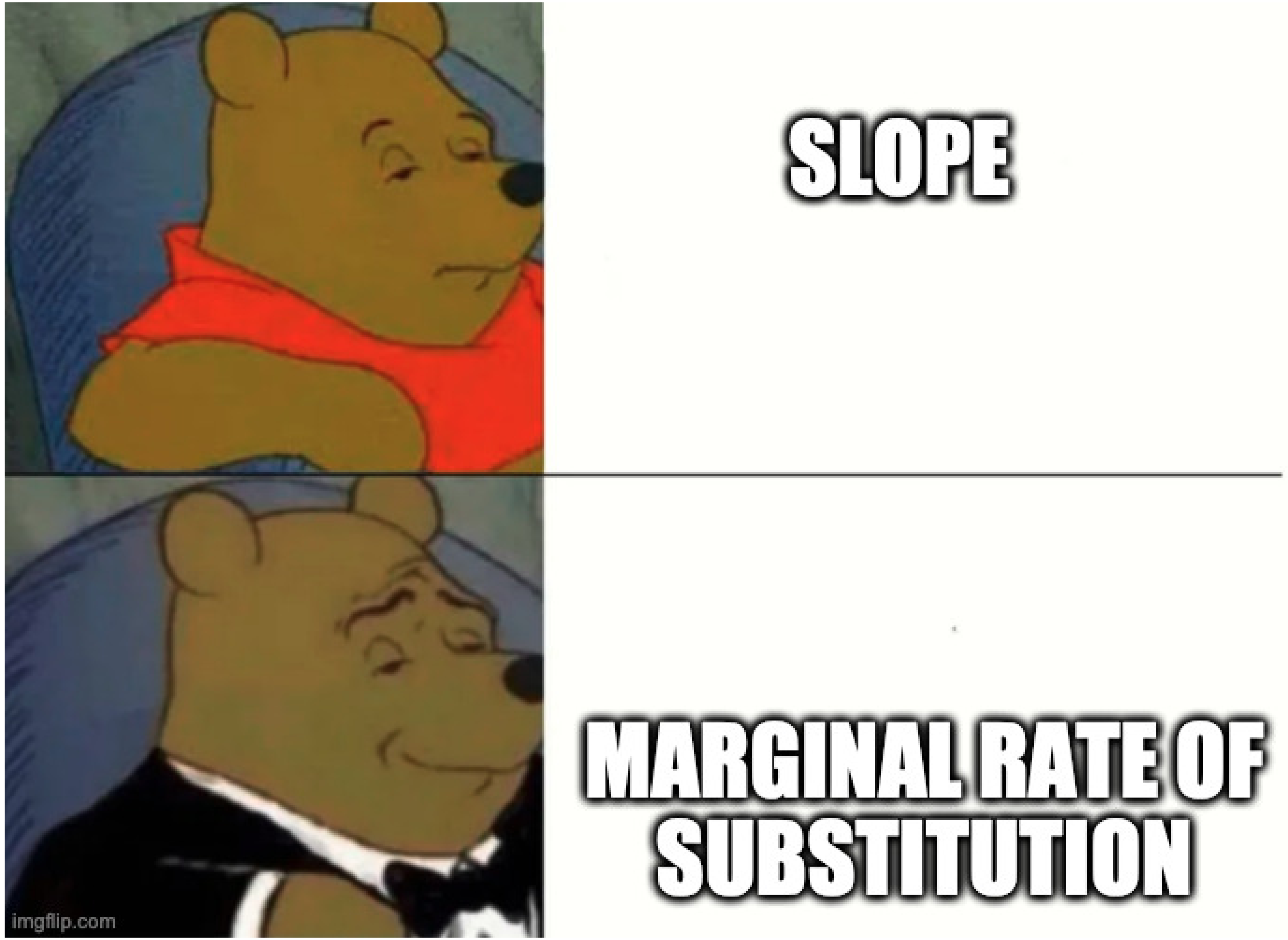
- If I find another apt with *1 fewer friend* nearby, how many *more* ft^2 would you need to keep you *satisfied?*

- **Marginal Rate of Substitution (MRS)**: rate at which you trade away one good for more of the other and remain *indifferent*

- Think of this as the **relative value** you place on good x:

> "I am willing to give up (MRS) units of y to consume 1 more unit of x and stay satisfied."

Marginal Rate of Substitution II



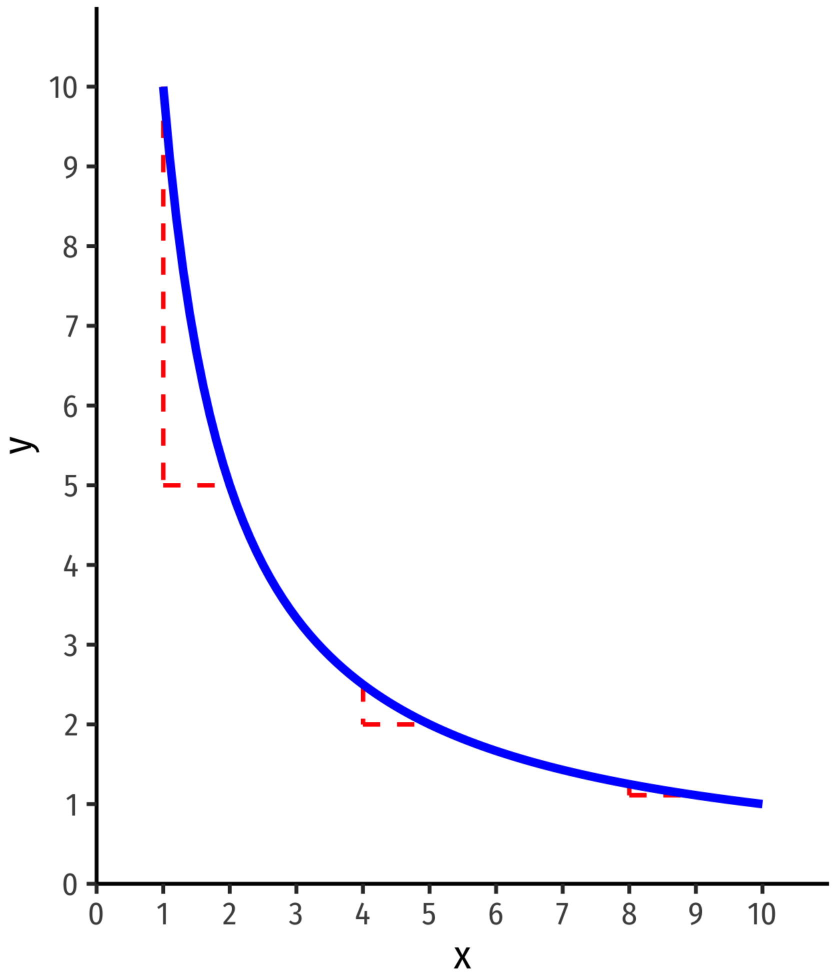
Marginal Rate of Substitution II

- MRS $=$ **slope of the indifference curve**

$$MRS_{x,y} = -\frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

- Amount of y given up for 1 more x

- Note: slope (MRS) changes along the curve!



MRS vs. Budget Constraint Slope

- Budget constraint (slope) from before measured the **market's** tradeoff between x and y based on market prices

- **MRS** here measures your **personal** evaluation of x vs. y based on your preferences

- Foreshadowing: what if these two rates are *different*? Are you truly optimizing?

Utility

So Where are the Numbers?

- Long ago (1890s), utility considered a real, measurable, cardinal scale†
- Utility thought to be lurking in people's brains
  - Could be understood from first principles: calories, water, warmth, etc
- Obvious problems

† "Neuroeconomics" & cognitive scientists are re-attempting a scientific approach to measure utility

Utility Functions?

- More plausibly **infer people's preferences from their actions**!
  - "Actions speak louder than words"
- **Principle of Revealed Preference**: if a person chooses x over y, and both are affordable, then they must prefer $x \succeq y$
- Flawless? Of course not. But extremely useful approximation!
  - People tend not to leave money on the table

Utility Functions!

- A **utility function** $u(\cdot)^{\dagger}$ *represents* preference relations $(\succ, \prec, \sim)$

- Assign utility numbers to bundles, such that, for any bundles a and b:

$$a \succ b \iff u(a) > u(b)$$

† The $\cdot$ is a placeholder for whatever goods we are considering (e.g. x, y, burritos, lattes, etc)

Utility Functions, Pural I

Example: Imagine three alternative bundles of (x, y):

$$a = (1, 2)$$
$$b = (2, 2)$$
$$c = (4, 3)$$

- Let $u(\cdot)$ assign each bundle a utility of:

$u(\cdot)$
$u(a) = 1$
$u(b) = 2$
$u(c) = 3$

- **Does this mean that bundle c is 3 times the utility of a?**

Utility Functions, Pural II

Example: Imagine three alternative bundles of (x, y):

$$a = (1, 2)$$
$$b = (2, 2)$$
$$c = (4, 3)$$

- Now consider a *2nd* function $v(\cdot)$:

$u(\cdot)$	$v(\cdot)$
$u(a) = 1$	$v(a) = 3$
$u(b) = 2$	$v(b) = 5$
$u(c) = 3$	$v(c) = 7$

Utility Functions, Pural III

- Utility numbers have an **ordinal** meaning only, **not cardinal**

- Both are valid utility functions:[†]

    - $u(c) > u(b) > u(a)$ ✅
    - $v(c) > v(b) > v(a)$ ✅
    - because $c \succ b \succ a$

- **Only the ranking of utility numbers matters!**

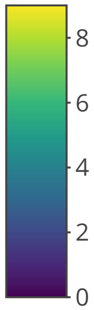
[†] See the Mathematical Appendix in Today's Class Page for why.

Utility Functions and Indifference Curves I

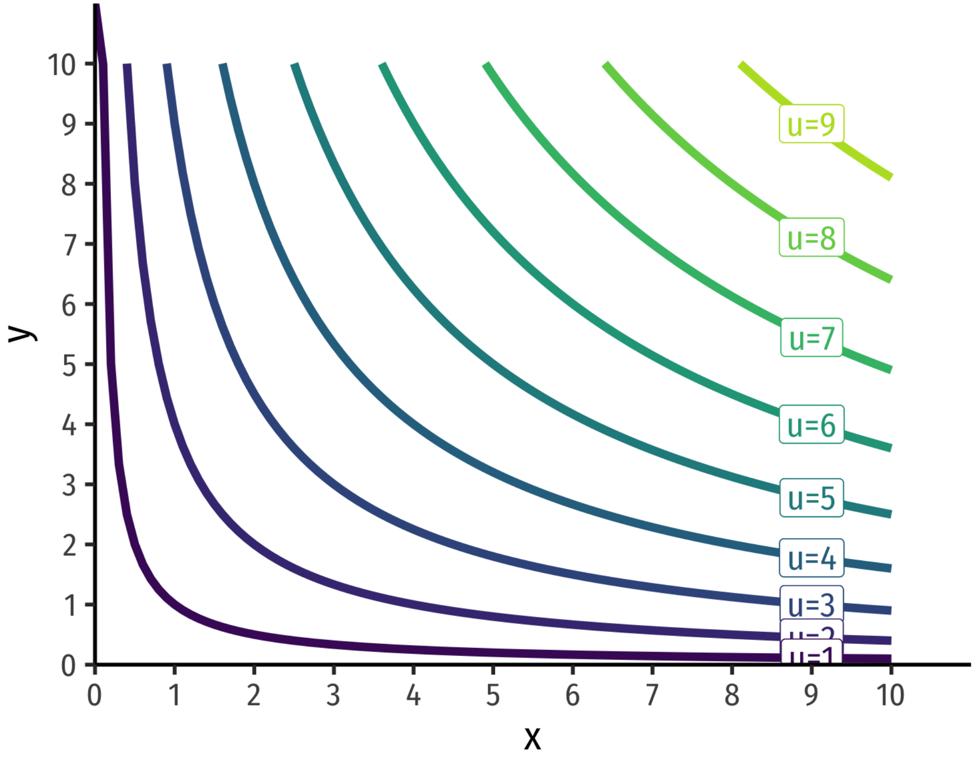
- Two tools to represent preferences: **indifference curves** and **utility functions**
- Indifference curve: all **equally preferred** bundles $\iff$ **same utility level**
- Each indifference curve represents one level (or contour) of utility surface (function)

Utility Functions and Indifference Curves II

3-D Utility Function: $u(x, y) = \sqrt{xy}$



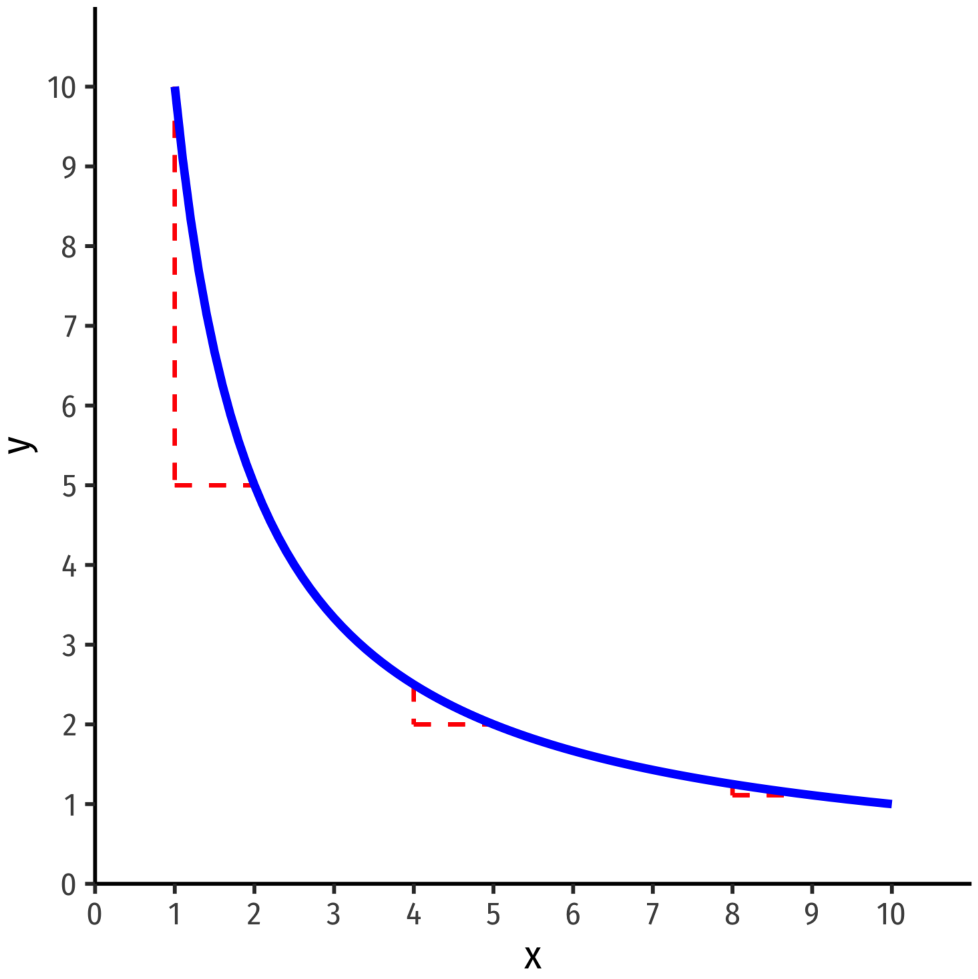
2-D Indifference Curve Contours: $y = \frac{u^2}{x}$



Marginal Utility

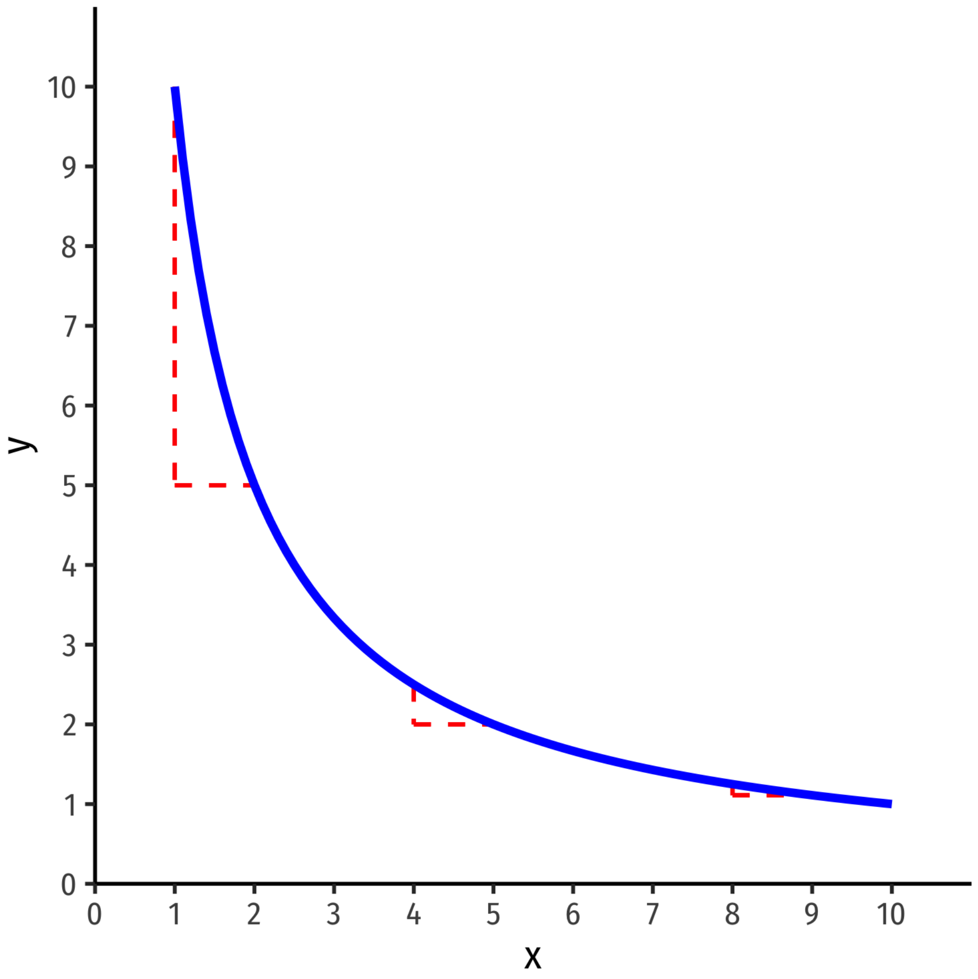
MRS and Marginal Utility I

- Recall: **marginal rate of substitution** $MRS_{x,y}$ is slope of the indifference curve
    - Amount of y given up for 1 more x
- How to calculate MRS?
    - Recall it changes (not a straight line)!
    - We can calculate it using something from the **utility function**



MRS and Marginal Utility II

- **Marginal utility**: change in utility from a marginal increase in consumption

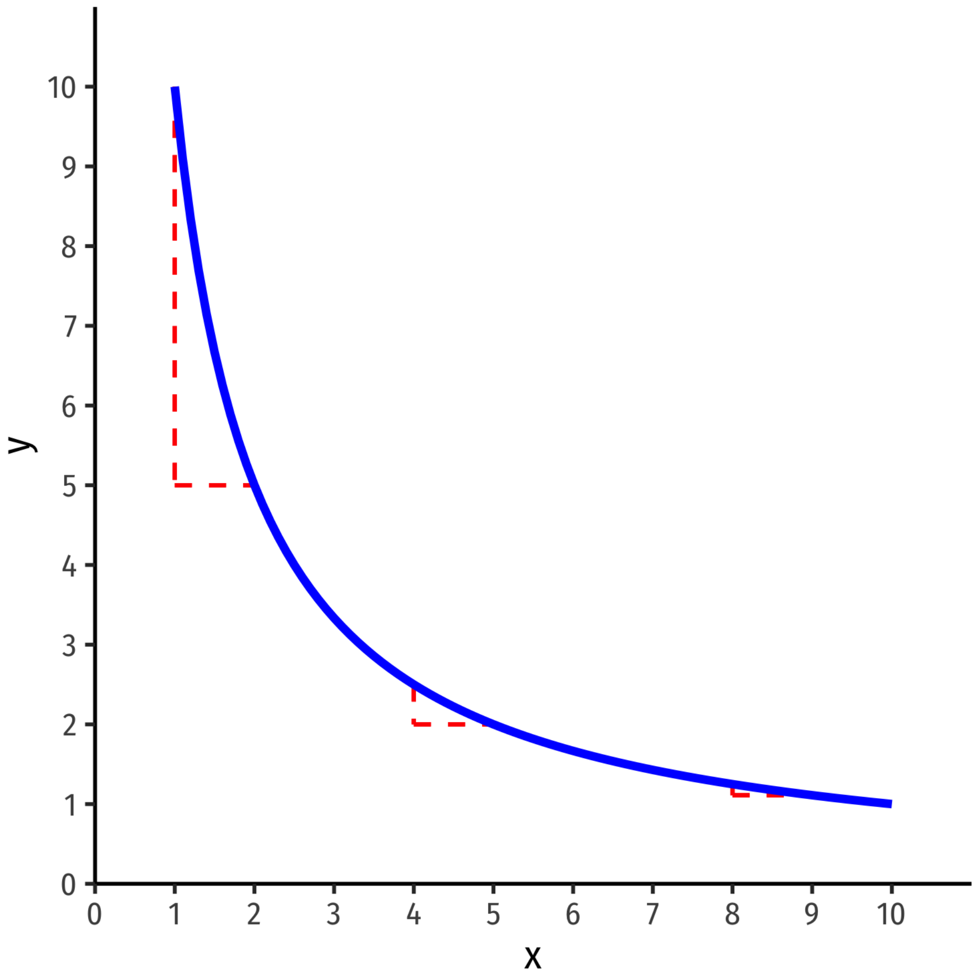


MRS and Marginal Utility II

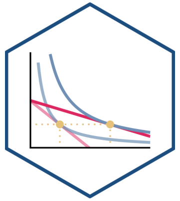
- **Marginal utility**: change in utility from a marginal increase in consumption

Marginal utility of x:

$$MU_x = \frac{\Delta u(x,y)}{\Delta x}$$



MRS and Marginal Utility II



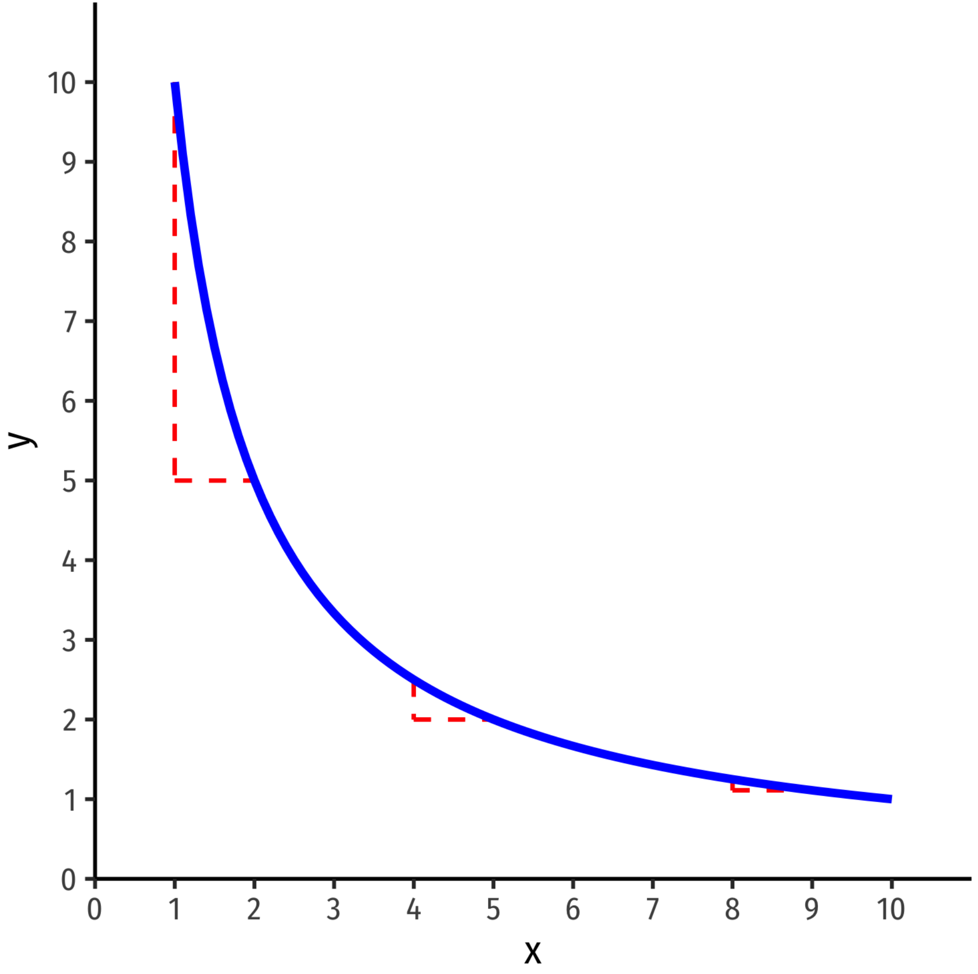
- **Marginal utility**: change in utility from a marginal increase in consumption

Marginal utility of x:

$$MU_x = \frac{\Delta u(x,y)}{\Delta x}$$

Marginal utility of y:

$$MU_y = \frac{\Delta u(x,y)}{\Delta y}$$



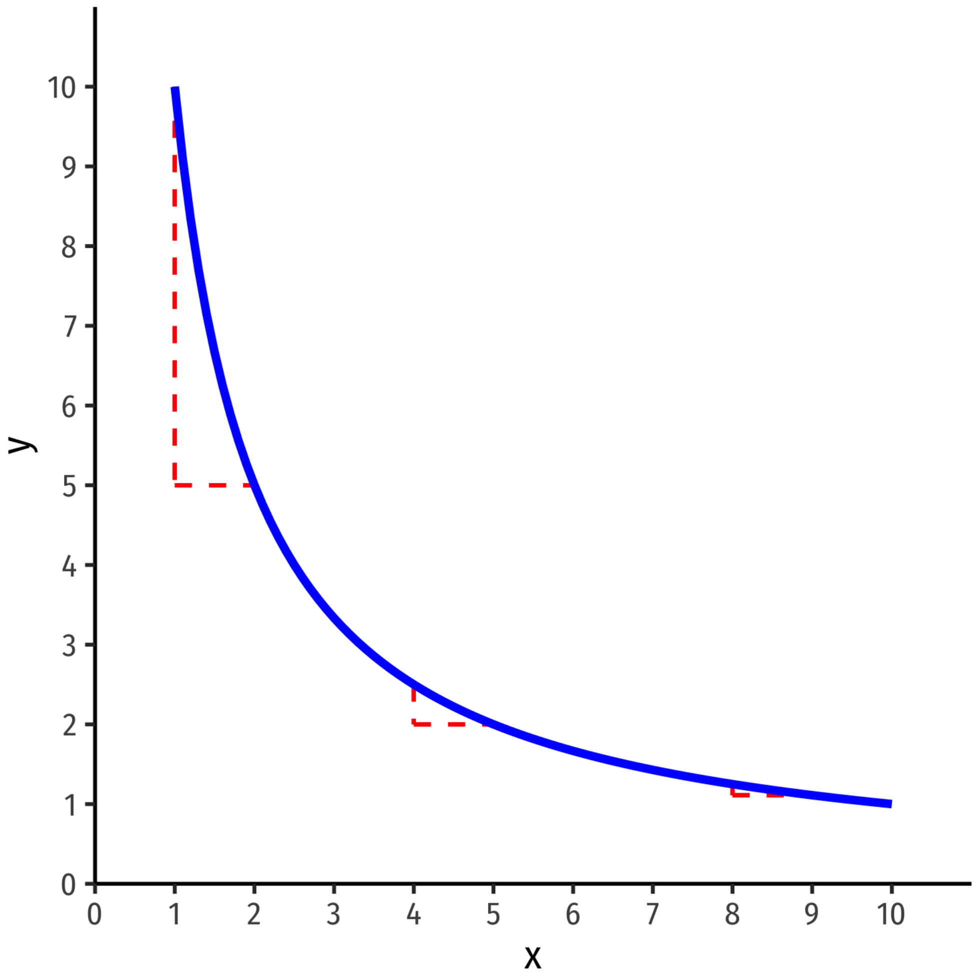
MRS and Marginal Utility II

- **Marginal utility**: change in utility from a marginal increase in consumption

- **Math (calculus)**: "*marginal*" $\iff$ "*derivative with respect to*"

$$MU_x = \frac{\partial \, u(x,y)}{\partial \, x}$$

- I will always derive marginal utility functions for you



MRS and Marginal Utility: Example

Example: For an example utility function:

$$u(x, y) = x^2 + y^3$$

- Marginal utility of x: $MU_x = 2x$
- Marginal utility of y: $MU_y = 3y^2$

- Again, I will always derive marginal utility functions for you

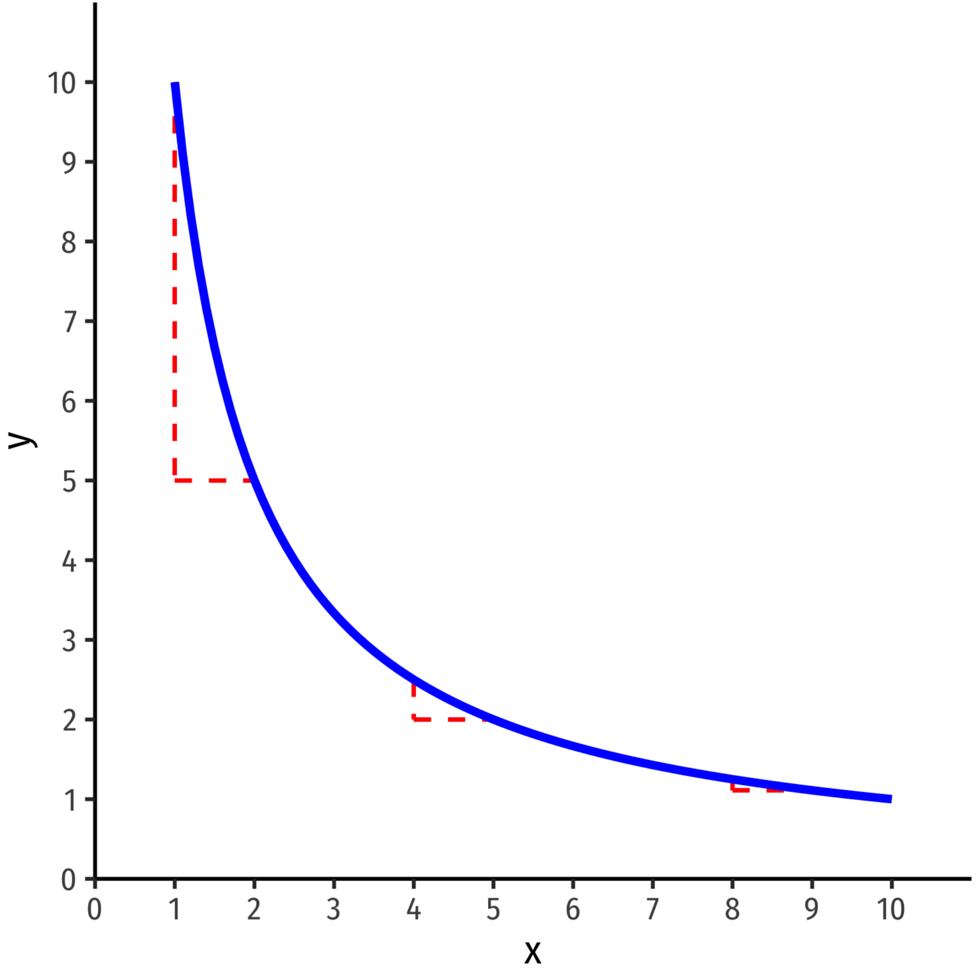
MRS Equation and Marginal Utility

- Relationship between MU and MRS:

$$\underbrace{\frac{\Delta y}{\Delta x}}_{MRS} = -\frac{MU_x}{MU_y}$$

- See proof in today's class notes

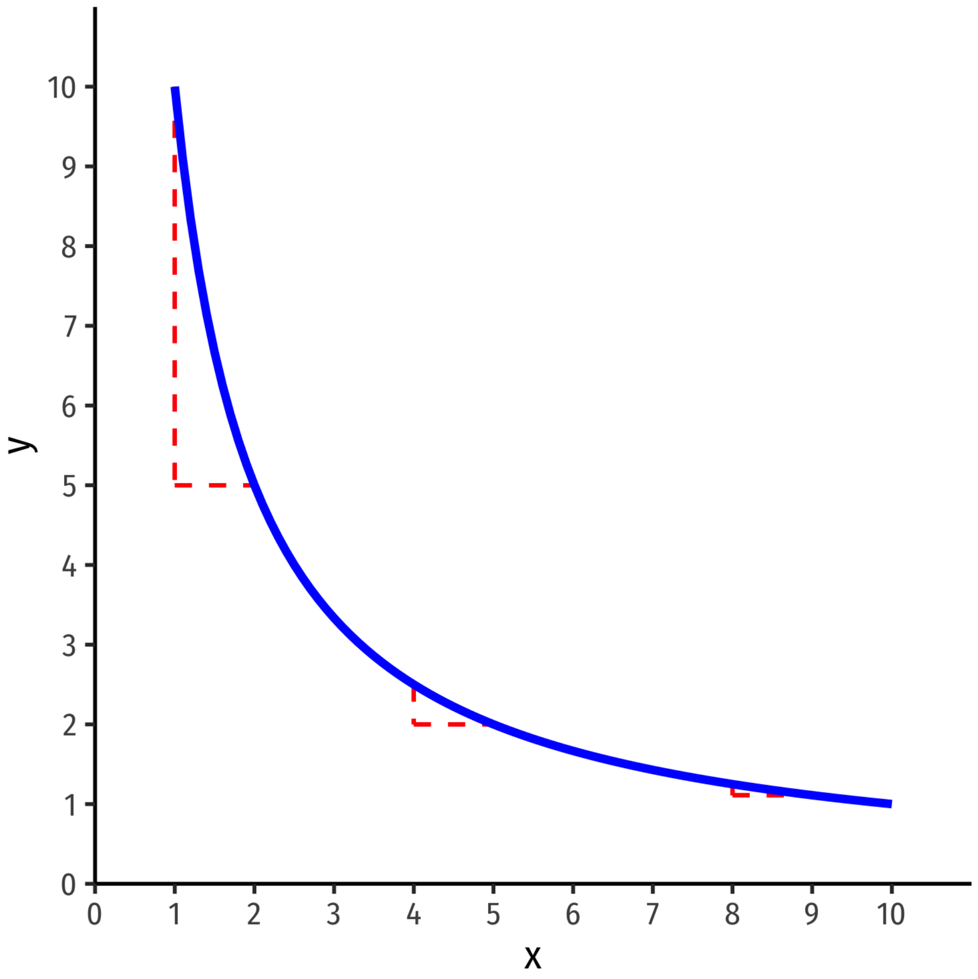
> "I am willing to give up $\frac{MU_x}{MU_y}$ units of y to consume 1 more unit of x and stay satisfied."



Important Insights About Value

> "I am willing to give up $\frac{MU_x}{MU_y}$ units of y to consume 1 more unit of x and stay satisfied."

- We can't measure MU's, but we *can* measure $MRS_{x,y}$ and infer the **ratio** of MU's!
  - **Example**: if $MRS_{x,y} = 5$, a unit of good x gives 5 times the marginal utility of good y at the margin



Important Insights About Value

- Value is **subjective**
    - Each of us has our own preferences that determine our ends or objectives
    - Choice is **forward looking**: a comparison of your **expectations** about opportunities
- **Preferences are not comparable across individuals**
    - Only individuals know what they give up at the moment of choice

Important Insights About Value

- Value inherently comes from the fact that we must make **tradeoffs**
  - Making one choice means *having to give up* pursuing others!
  - The choice we pursue at the moment must be worth the sacrifice of others! (i.e. highest marginal utility)

Diminishing Marginal Utility

The Law of Diminishing Marginal Utility: each marginal unit of a good consumed tends to provide less marginal utility than the previous unit, all else equal

- As you consume more x:
  - $\downarrow MU_x$
  - $\downarrow MRS_{x,y}$: willing to give up *fewer* units of y for x

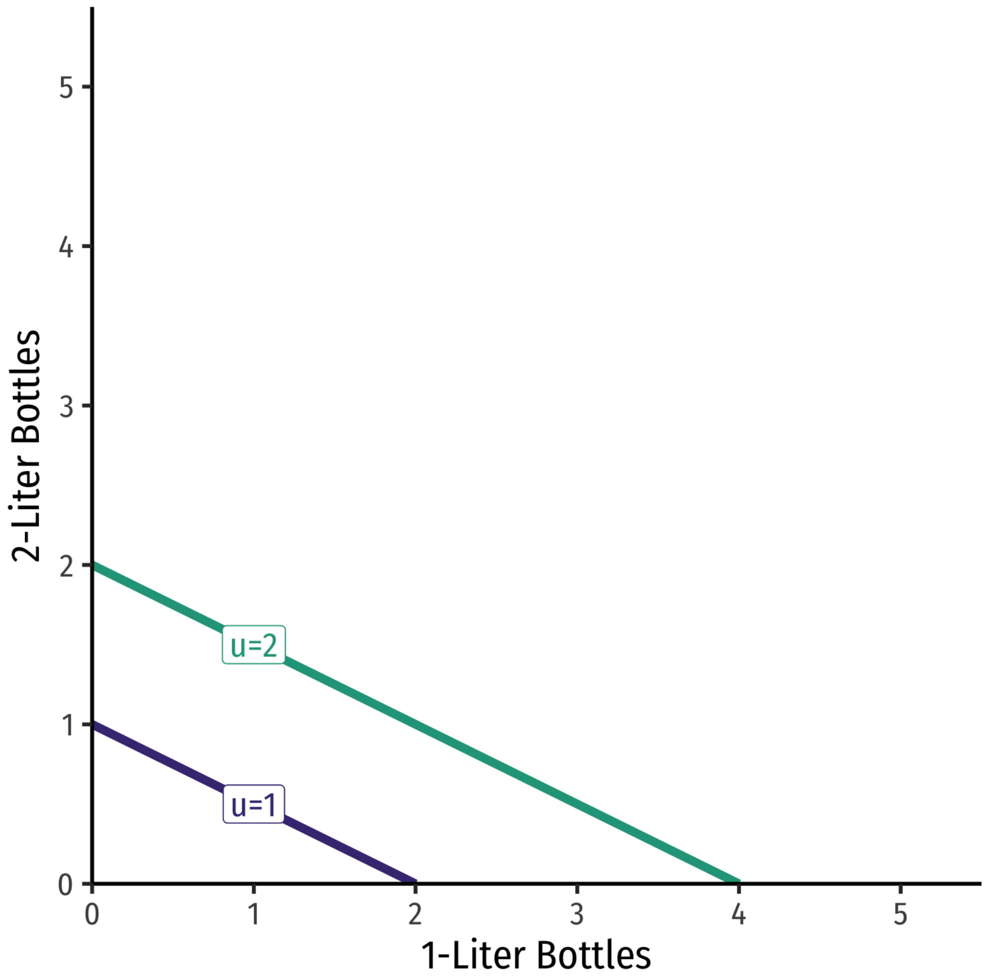
Special Case: Substitutes

> **Example**: Consider 1-Liter bottles of coke and 2-Liter bottles of coke

- Always willing to substitute between Two 1-L bottles for One 2-L bottle

- **Perfect substitutes**: goods that can be substituted at same fixed rate and yield same utility

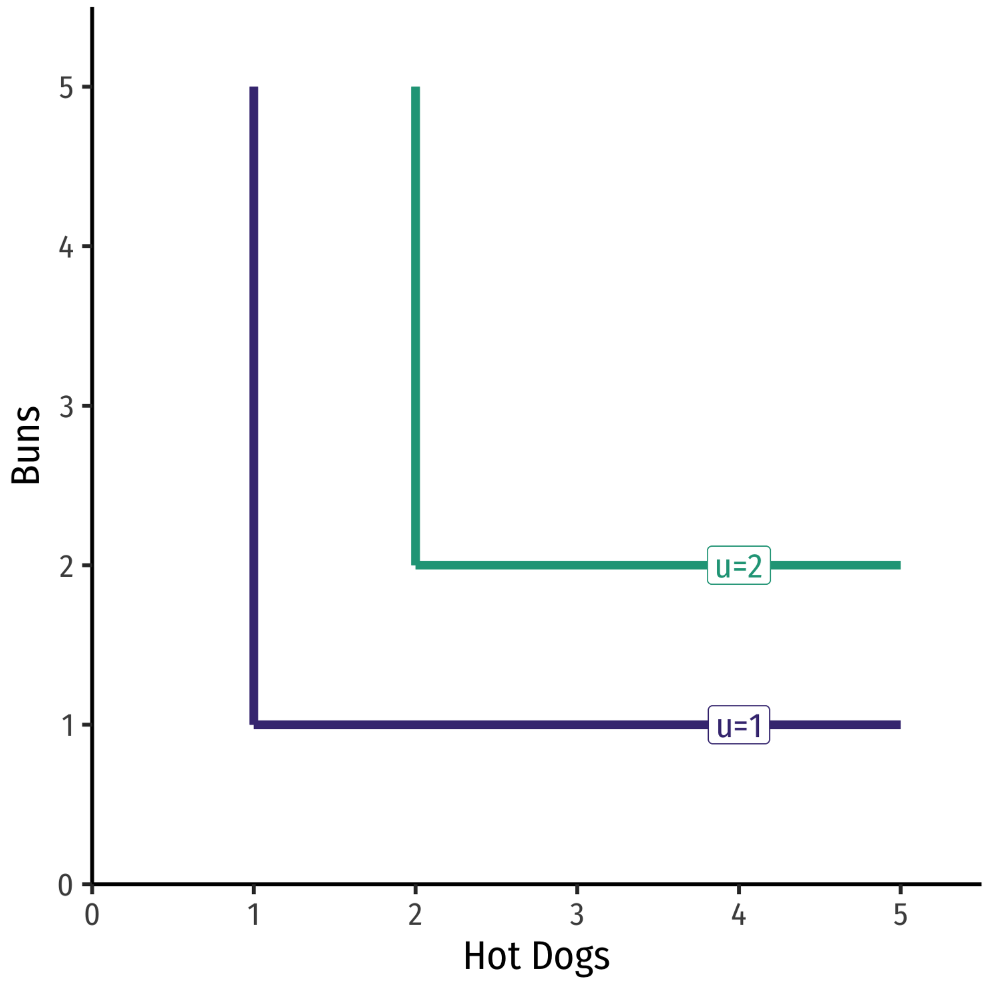
- $MRS_{1L,2L} = -0.5$ (a constant!)



Special Case: Complements

Example: Consider hot dogs and hot dog buns

- Always consume together in fixed proportions (in this case, 1 for 1)
- **Perfect complements**: goods that can be consumed together in same fixed proportion and yield same utility
- $MRS_{H,B} = ?$

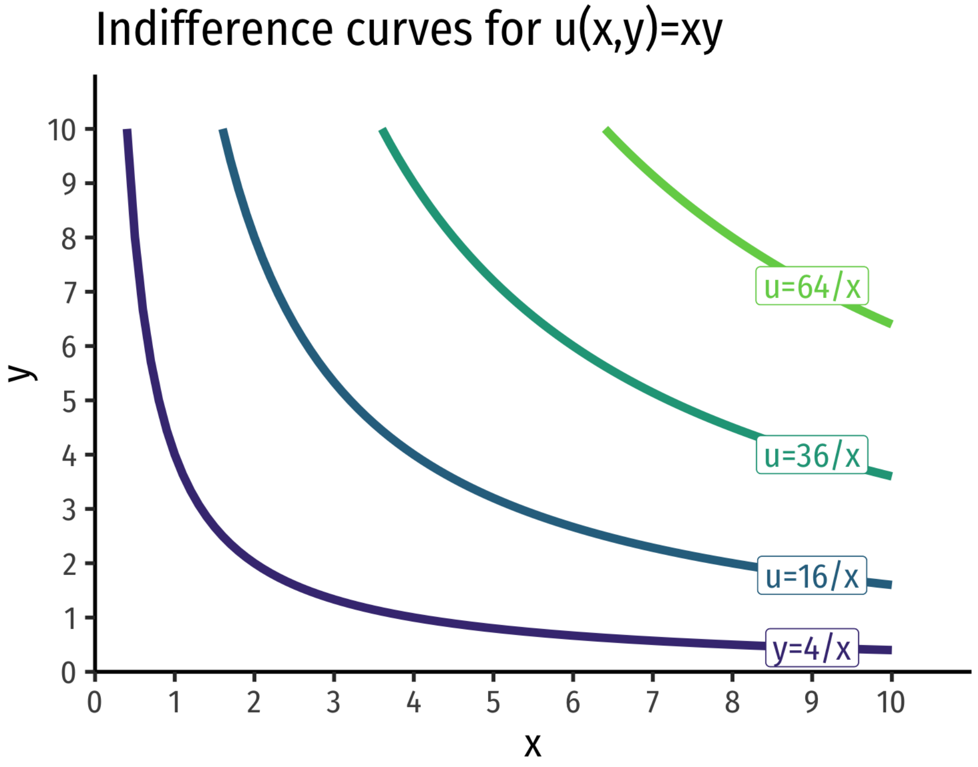


Cobb-Douglas Utility Functions

- A very common functional form in economics is **Cobb-Douglas**

$$u(x,y) = x^a y^b$$

- Extremely useful, you will see it often!
  - Lots of nice, useful properties (we'll see later)
  - See the appendix in [today's class page]



Practice

Example: Suppose you can consume apples (a) and broccoli (b), and earn utility according to:

$$u(a,b) = 2ab$$
$$MU_a = 2b$$
$$MU_b = 2a$$

1. Put a on the horizontal axis and b on the vertical axis. Write an equation for $MRS_{a,b}$.

2. Would you prefer a bundle of $(1,4)$ or $(2,2)$?

3. Suppose you are currently consuming 1 apple and 4 broccoli. a. How many units of broccoli are you willing to give up to eat 1 more apple and remain indifferent? b. How much *more* utility would you get if you were to eat 1 more apple?

4. Repeat question 3, but for when you are consuming 2 of each good.