2.5 — Short Run Profit Maximization

ECON 306 • Microeconomic Analysis • Spring 2023

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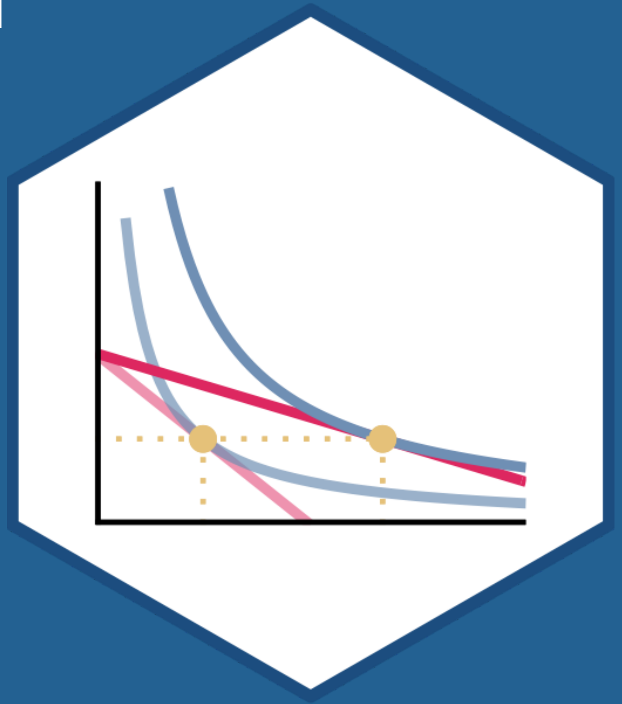
[ryansafner/microS23](https://github.com/ryansafner/microS23)

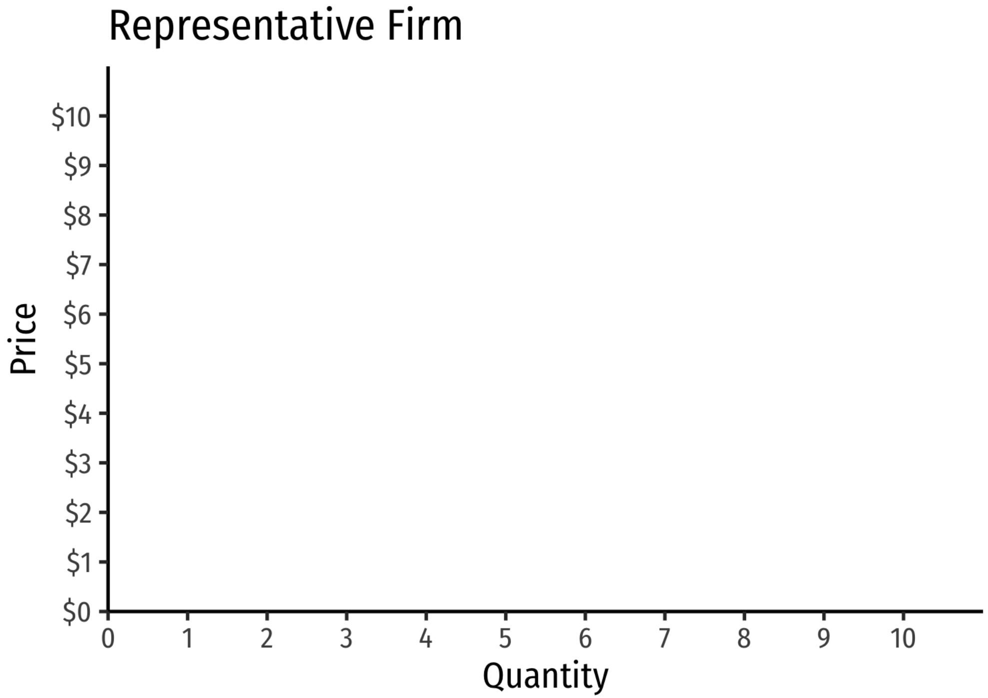
microS23.classes.ryansafner.com

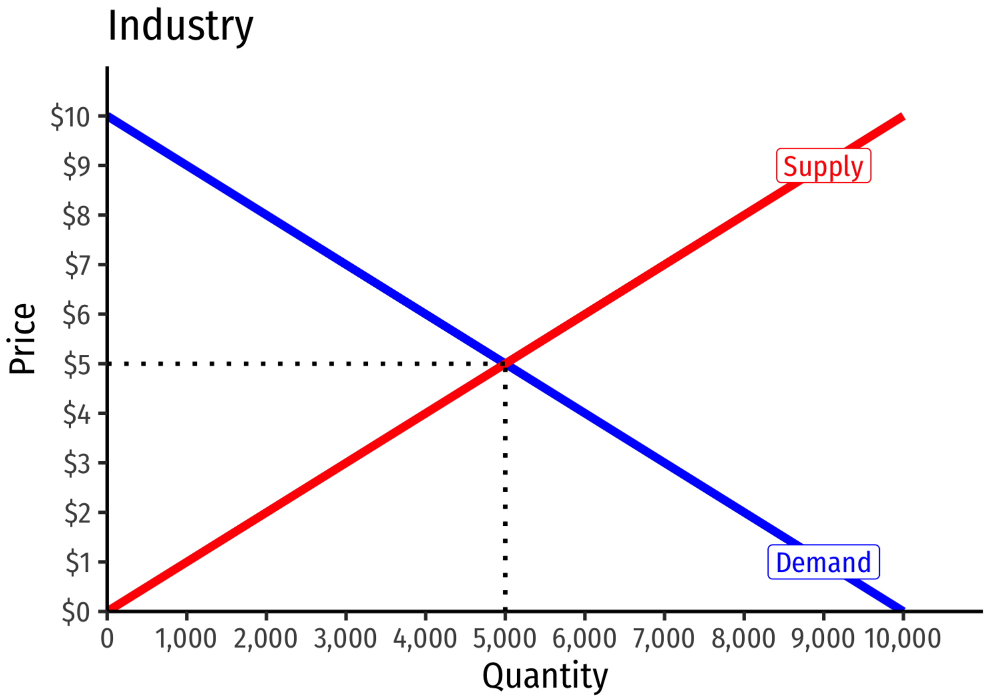


Outline

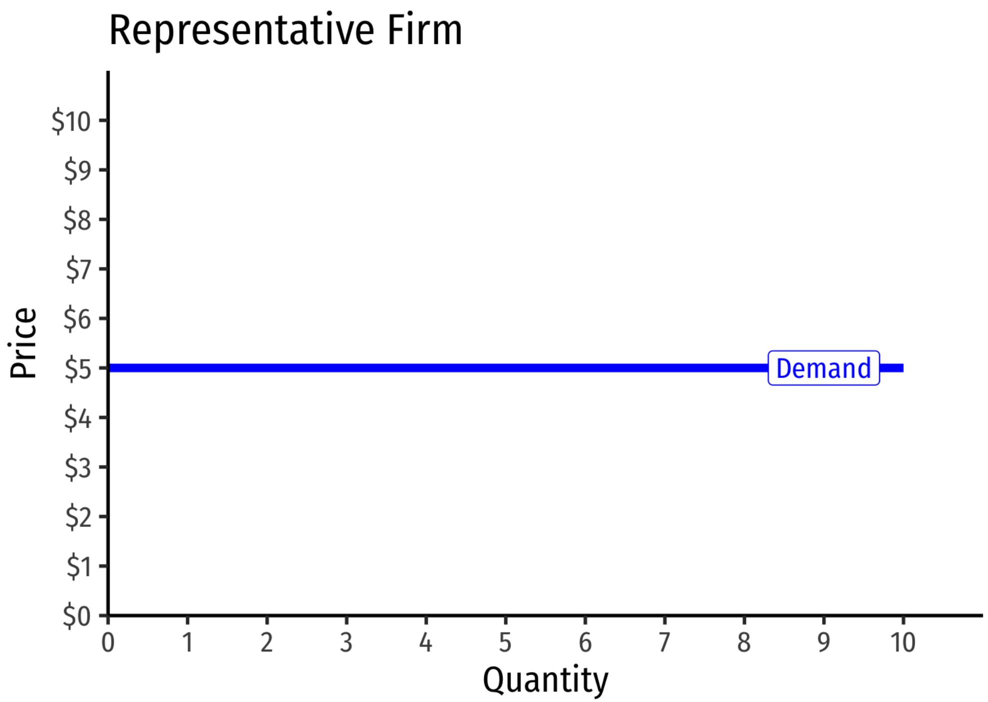
Revenues

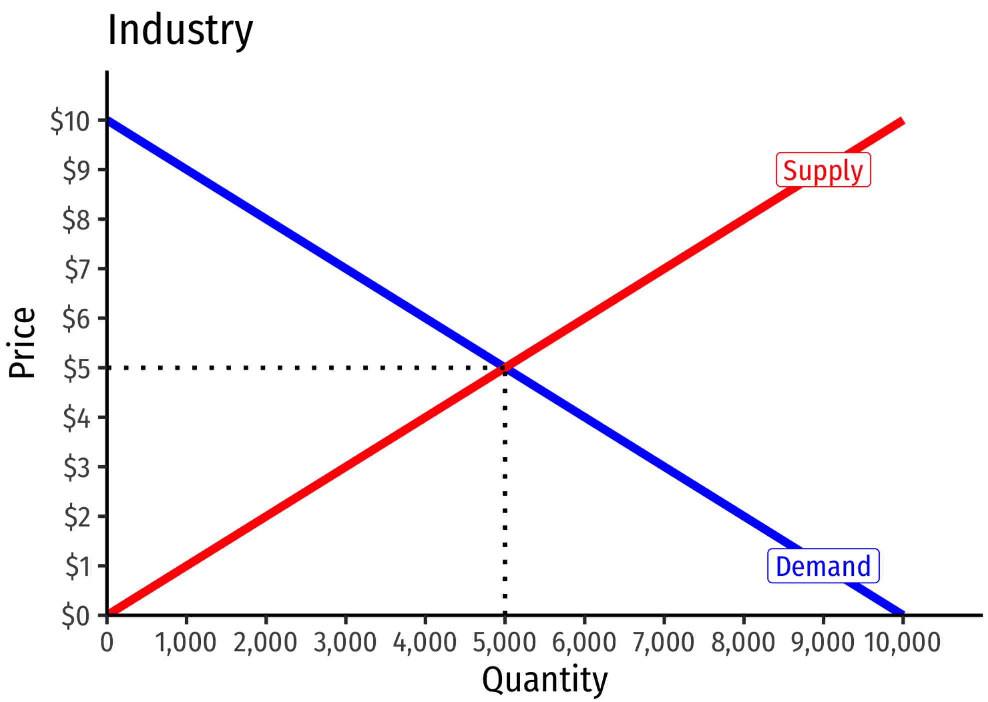
Revenues for Firms in *Competitive* Industries I





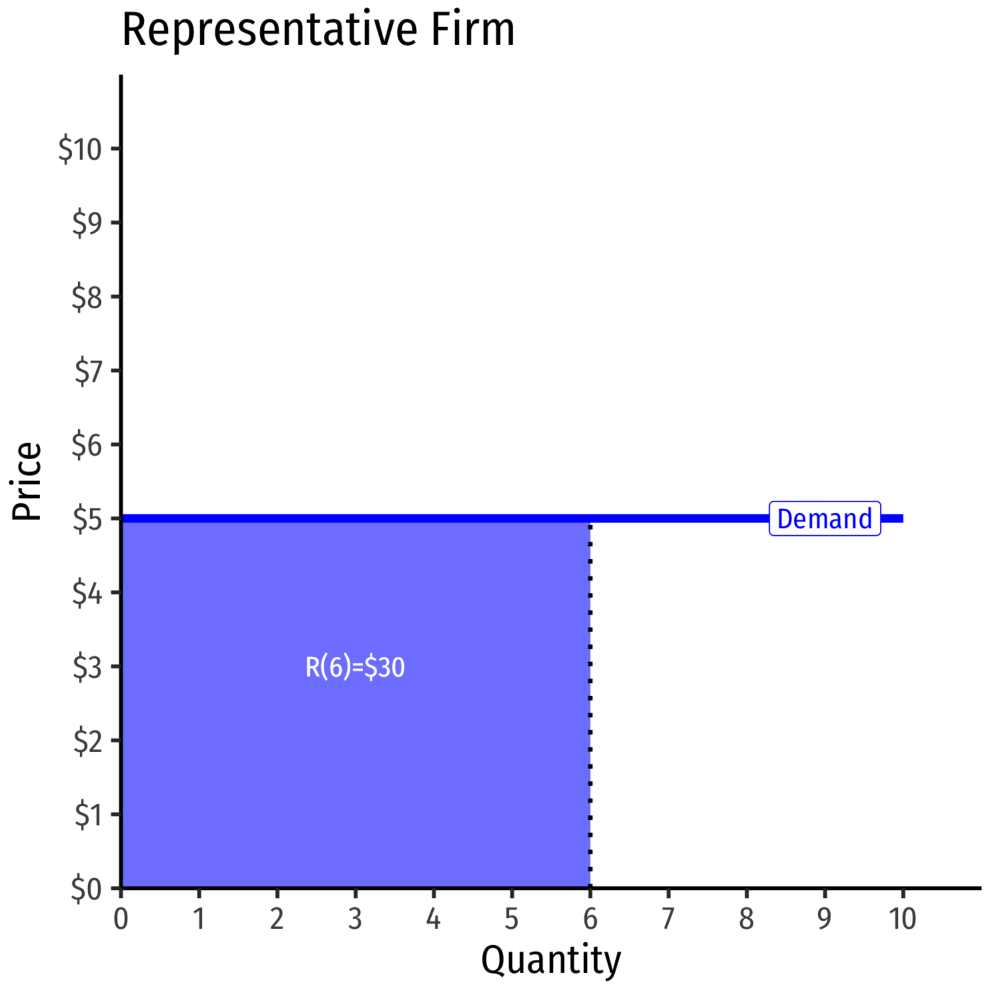
Revenues for Firms in *Competitive* Industries I





- Demand for a firm's product is **perfectly elastic** at the market price
- Where did the supply curve come from? You'll know today

Revenues for Firms in *Competitive* Industries II



- **Total Revenue** $R(q) = pq$

Average and Marginal Revenues

- **Average Revenue**: revenue per unit of output

$$AR(q) = \frac{R}{q}$$

  - $AR(q)$ is **by definition** equal to the price! (Why?)

- **Marginal Revenue**: change in revenues for each additional unit of output sold:

$$MR(q) = \frac{\Delta R(q)}{\Delta q}$$

  - Calculus: first derivative of the revenues function
  - **For a *competitive* firm (only), MR(q) = p, i.e. the price!**

Average and Marginal Revenues: Example

Example: A firm sells bushels of wheat in a very competitive market. The current market price is $10/bushel.

For the 1st bushel sold:

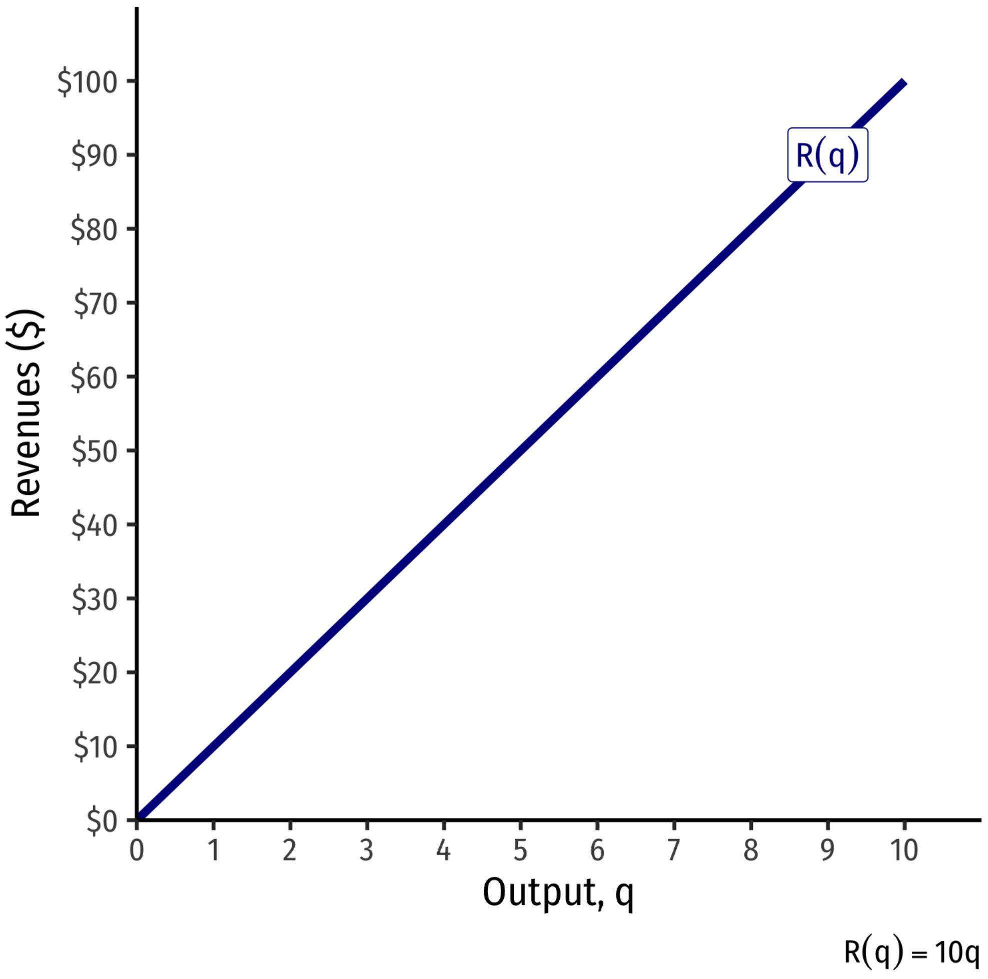
- What is the total revenue?
- What is the average revenue?

For the 2nd bushel sold:

- What is the total revenue?
- What is the average revenue?
- What is the marginal revenue?

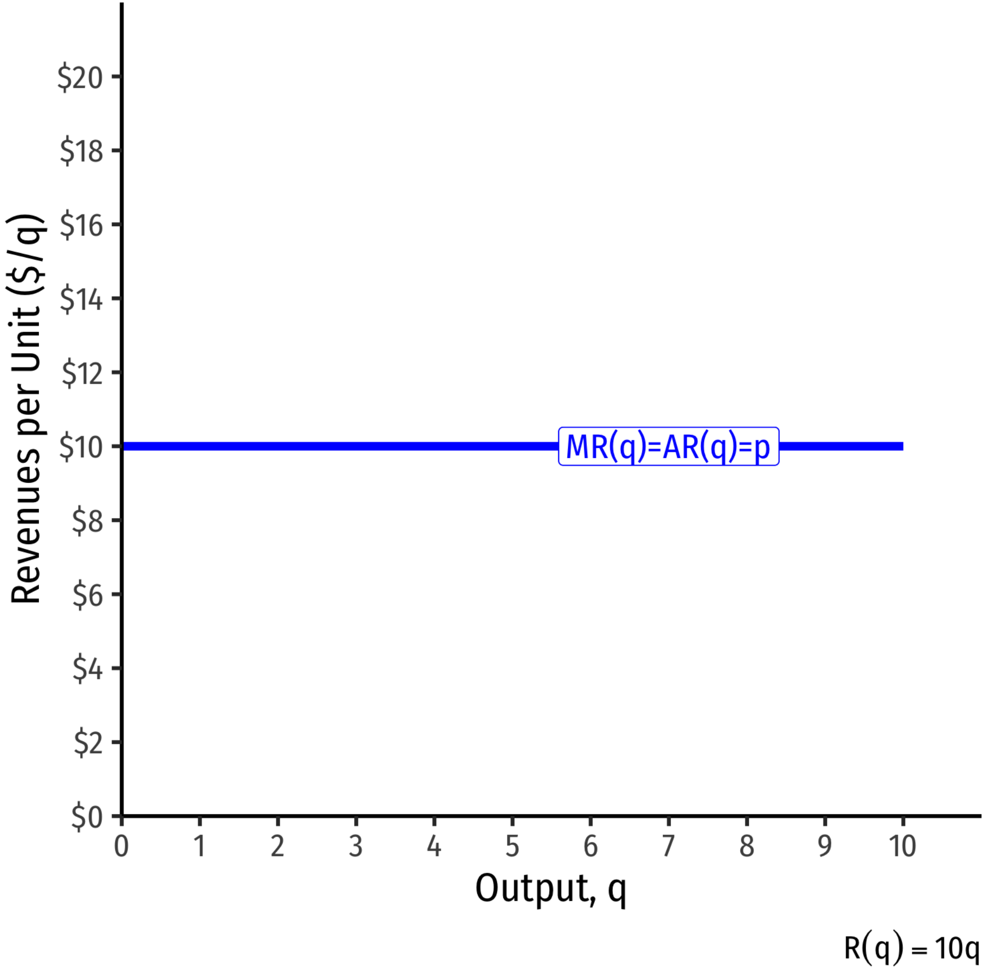
Total Revenue, Example: Visualized

q	$R(q)$
0	0
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100



Average and Marginal Revenue, Example: Visualized

q	$R(q)$	$AR(q)$	$MR(q)$
0	0	–	–
1	10	10	10
2	20	10	10
3	30	10	10
4	40	10	10
5	50	10	10
6	60	10	10
7	70	10	10
8	80	10	10
9	90	10	10
10	100	10	10



Profits

Recall: The Firm's Two Problems

1st Stage: firm's profit maximization problem:

1. **Choose:** < output >
2. **In order to maximize:** < profits >

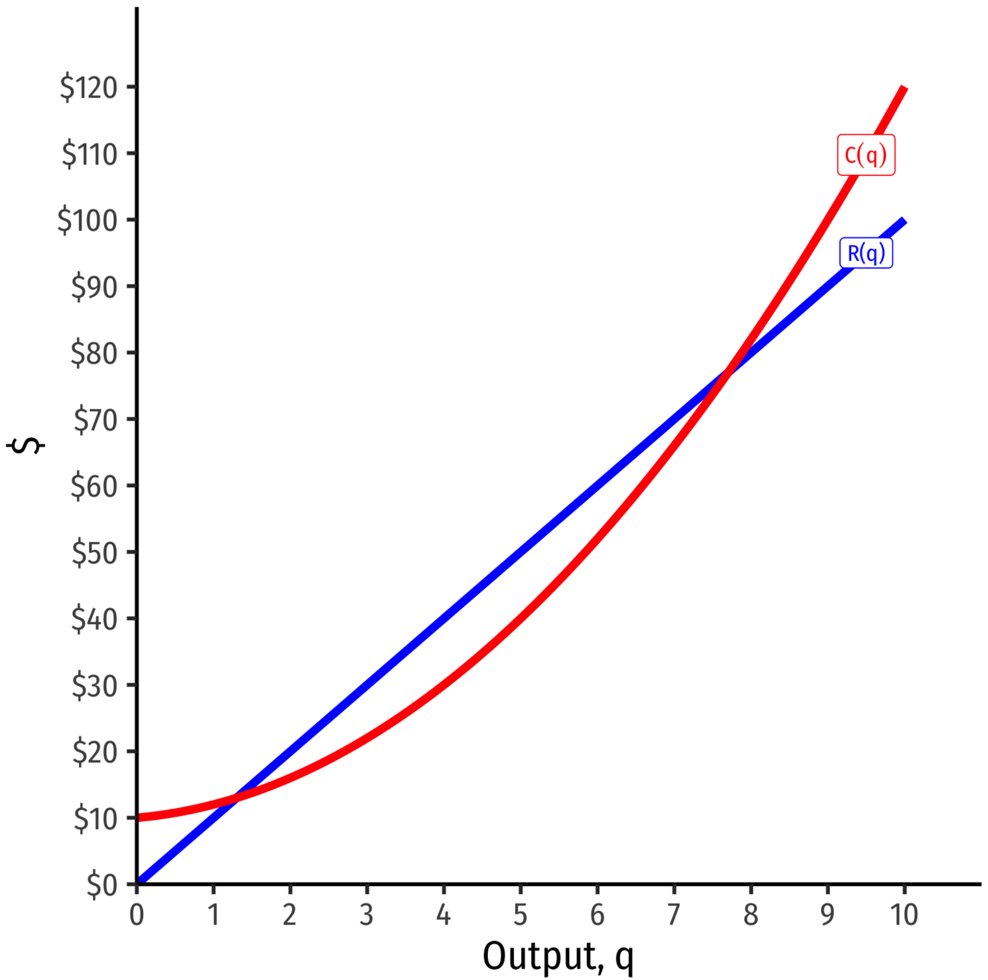
2nd Stage: firm's cost minimization problem:

1. **Choose:** < inputs >
2. **In order to *minimize*:** < cost >
3. **Subject to:** < producing the optimal output >

- Minimizing costs $\iff$ maximizing profits

Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$



Visualizing Total Profit As $R(q) - C(q)$

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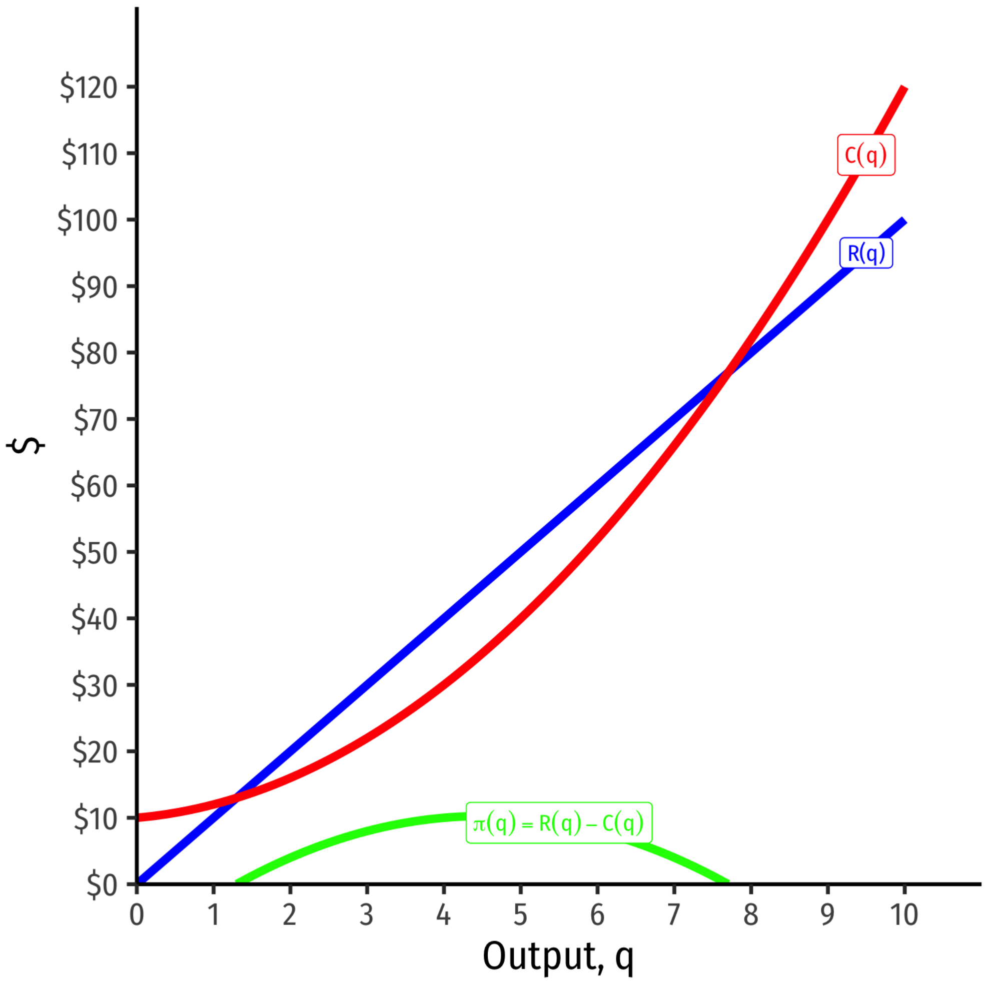
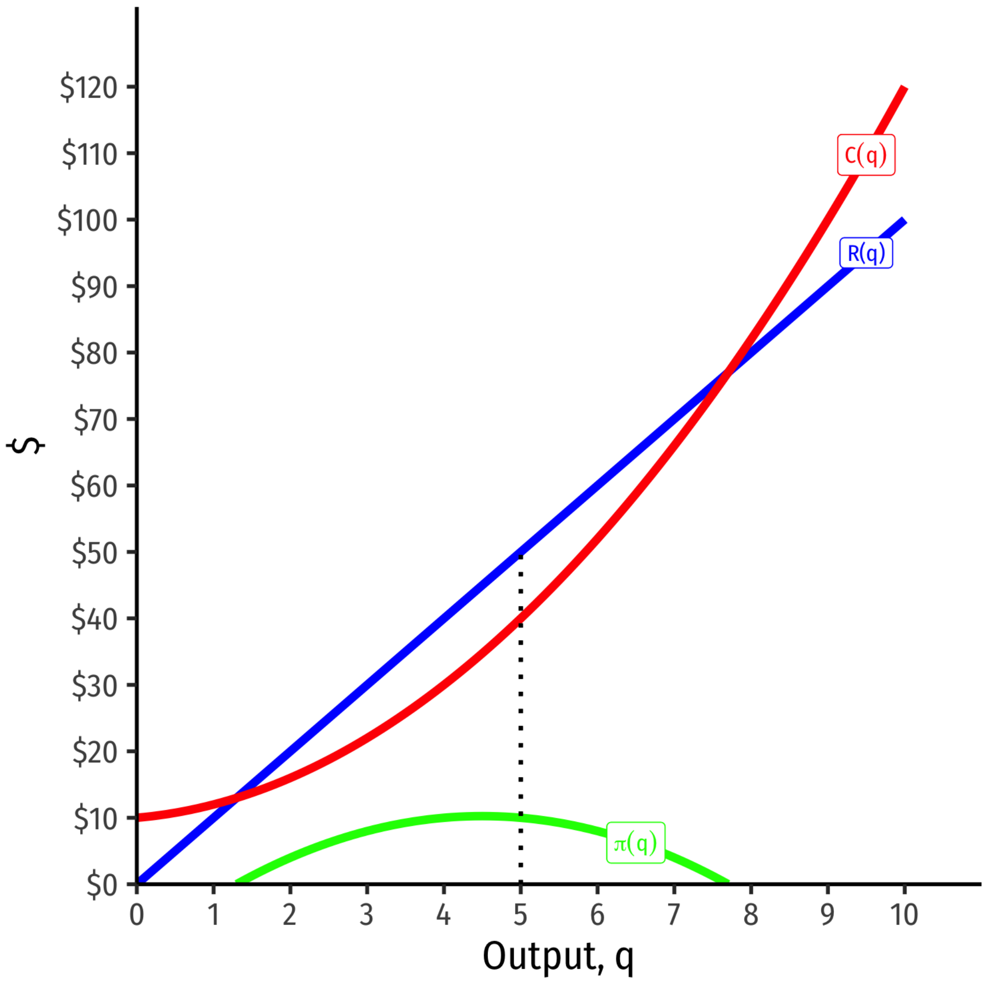


Visualizing Total Profit As $R(q) - C(q)$

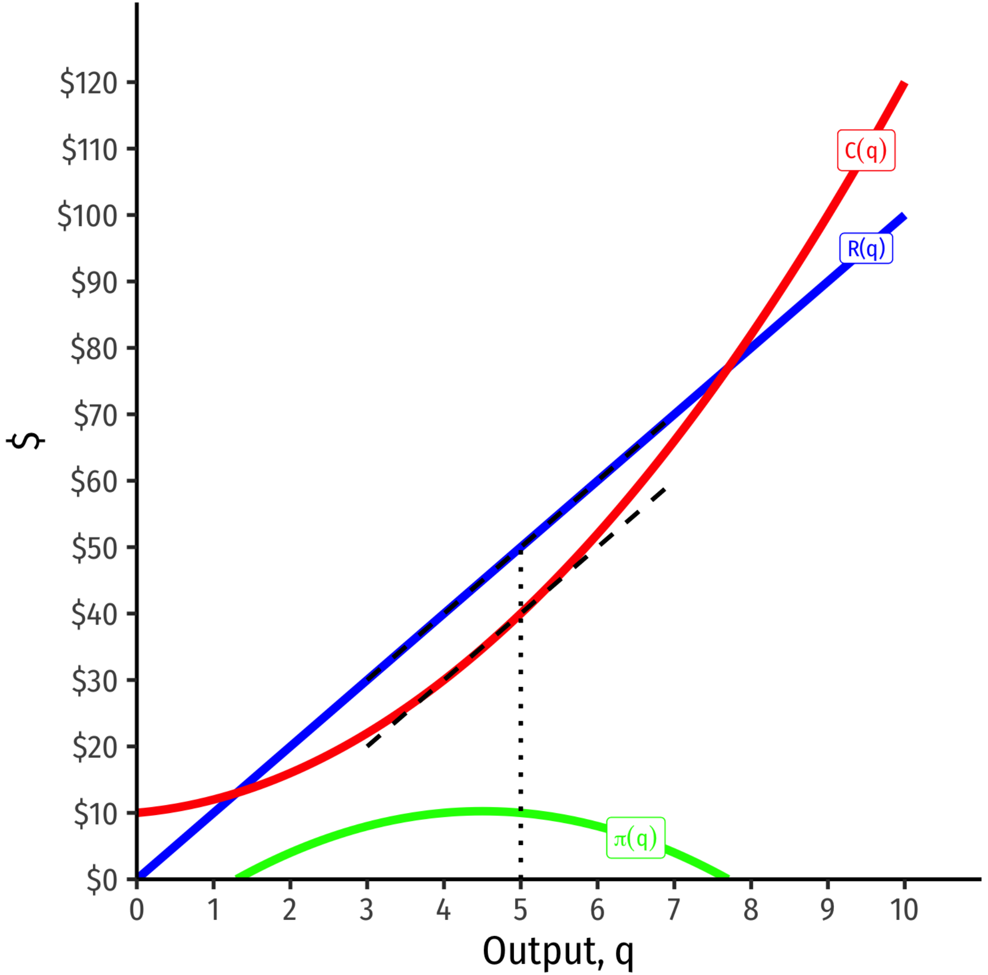
- $\pi(q) = R(q) - C(q)$
- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$



Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$
- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$
- Slopes must be equal:

$$MR(q) = MC(q)$$



Visualizing Total Profit As $R(q) - C(q)$

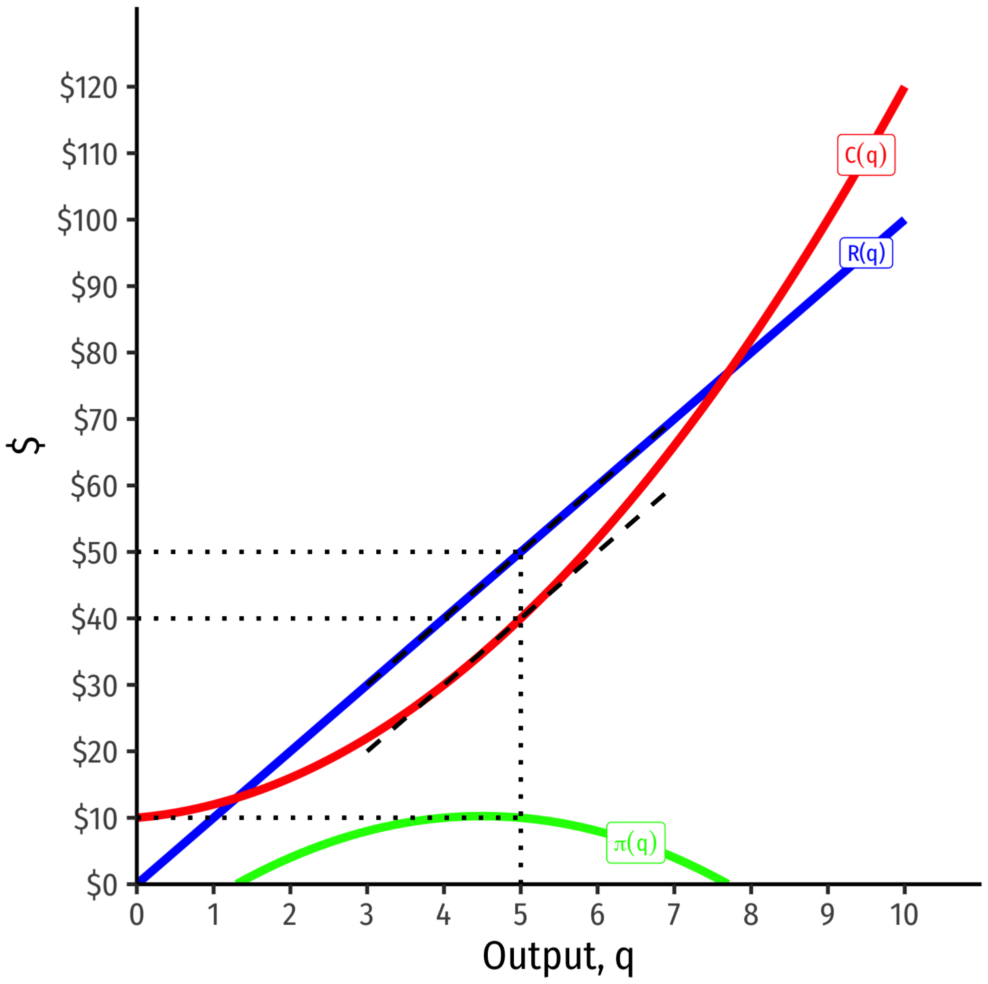
- $\pi(q) = R(q) - C(q)$

- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$

- Slopes must be equal:

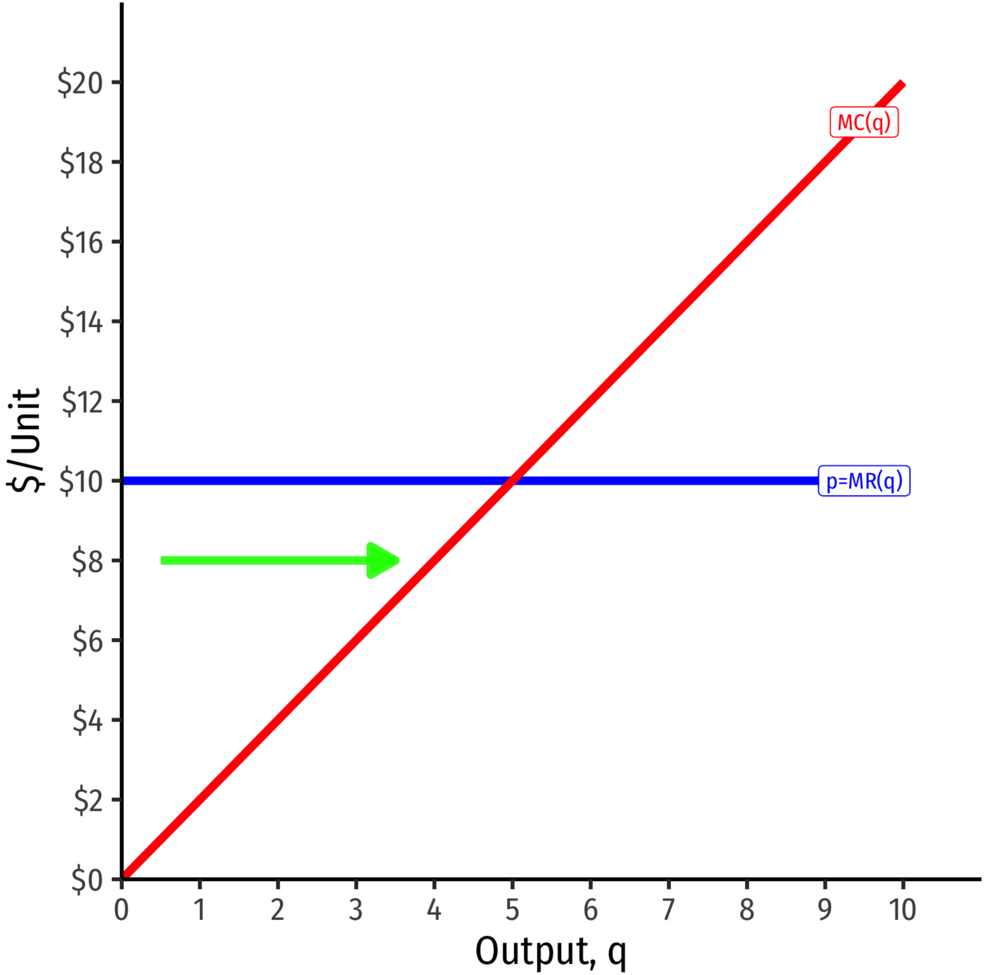
$$MR(q) = MC(q)$$

- At $q^* = 5$:
  - $R(q) = 50$
  - $C(q) = 40$
  - $\pi(q) = 10$



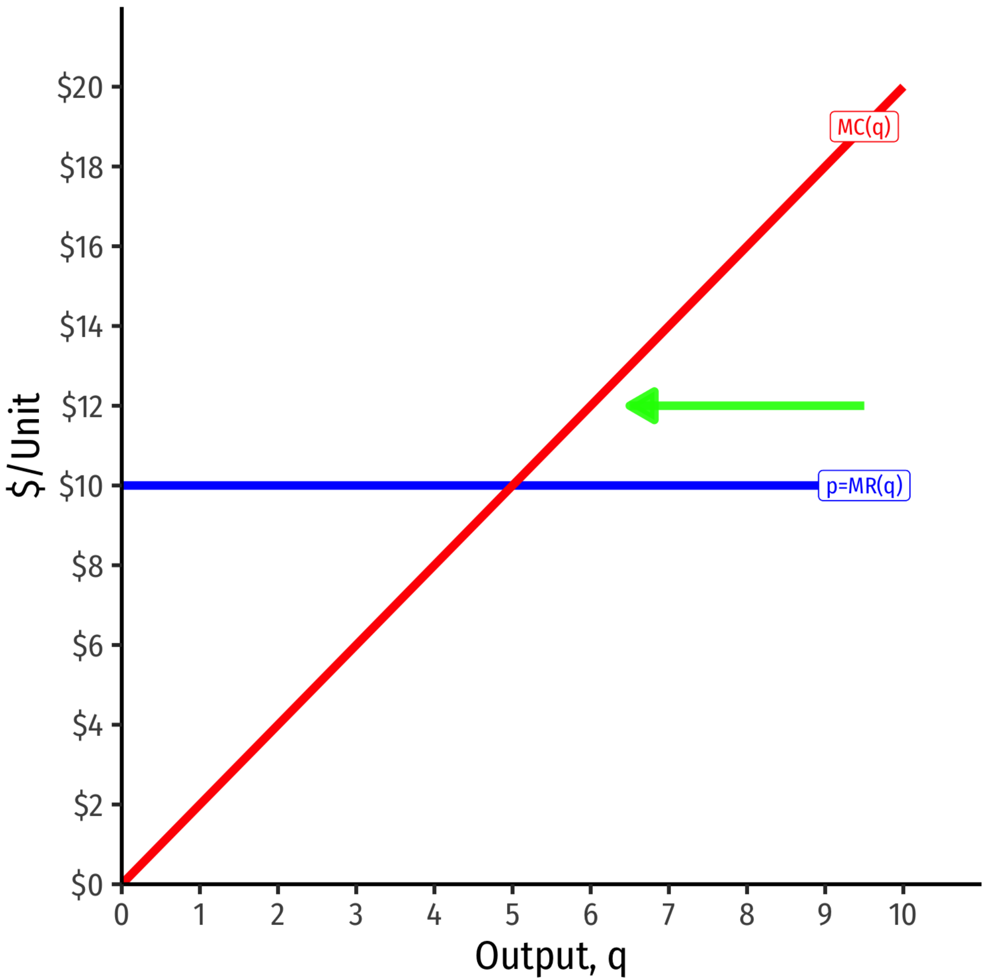
Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

- At low output $q < q^*$, can increase π by producing *more*: $MR(q) > MC(q)$



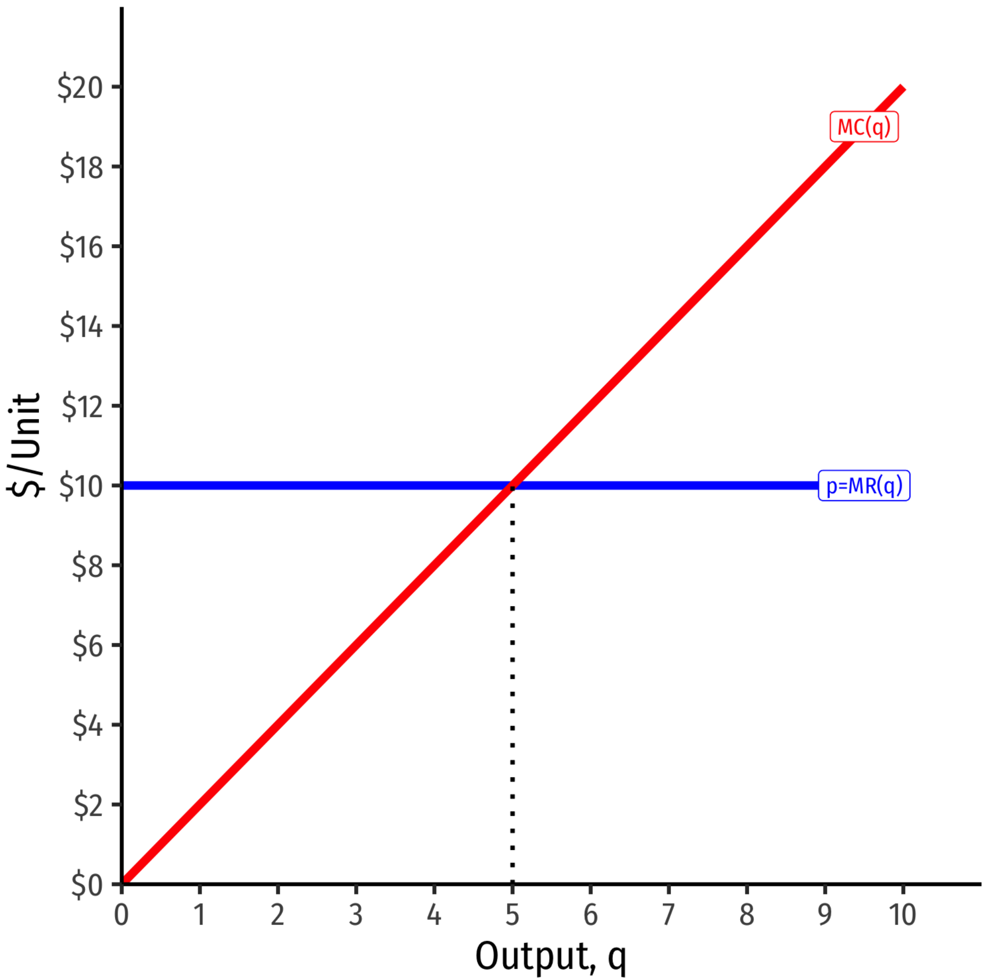
Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

- At high output $q > q^*$, can increase π by producing *less*: $MR(q) < MC(q)$



Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

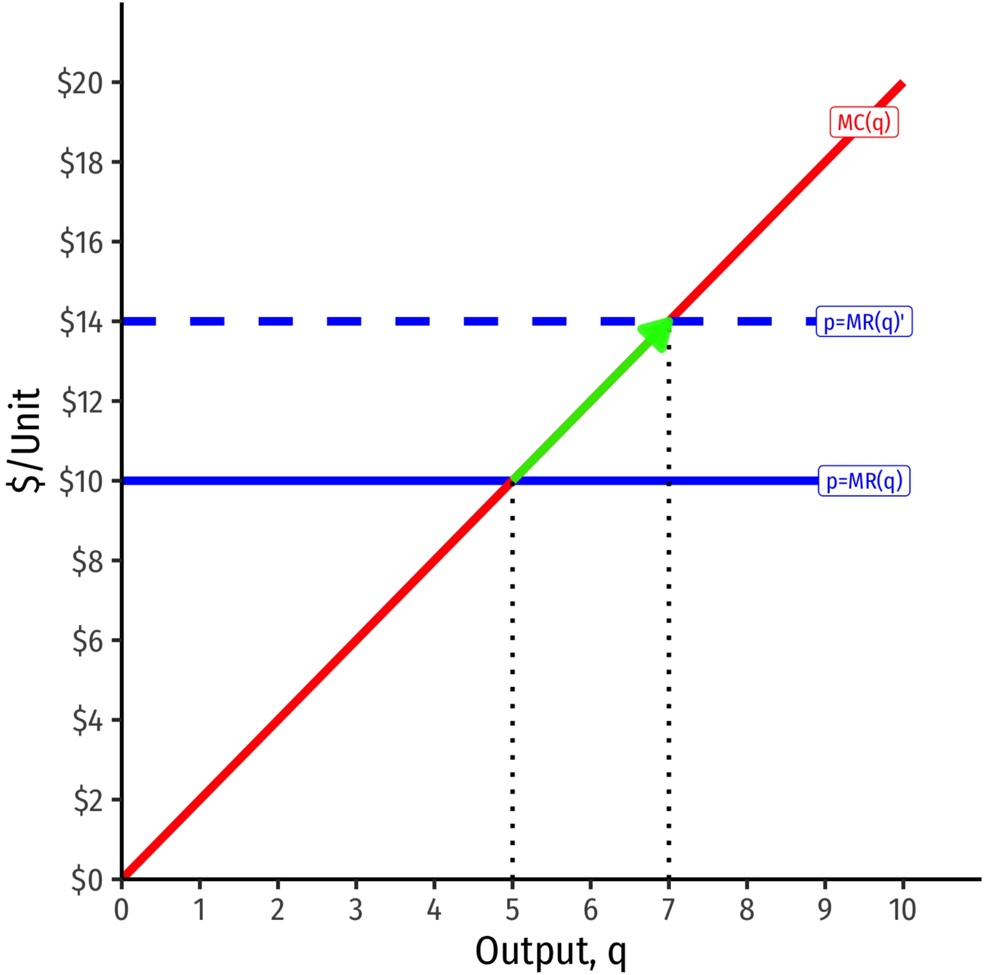
- π is *maximized* where $MR(q) = MC(q)$



Comparative Statics

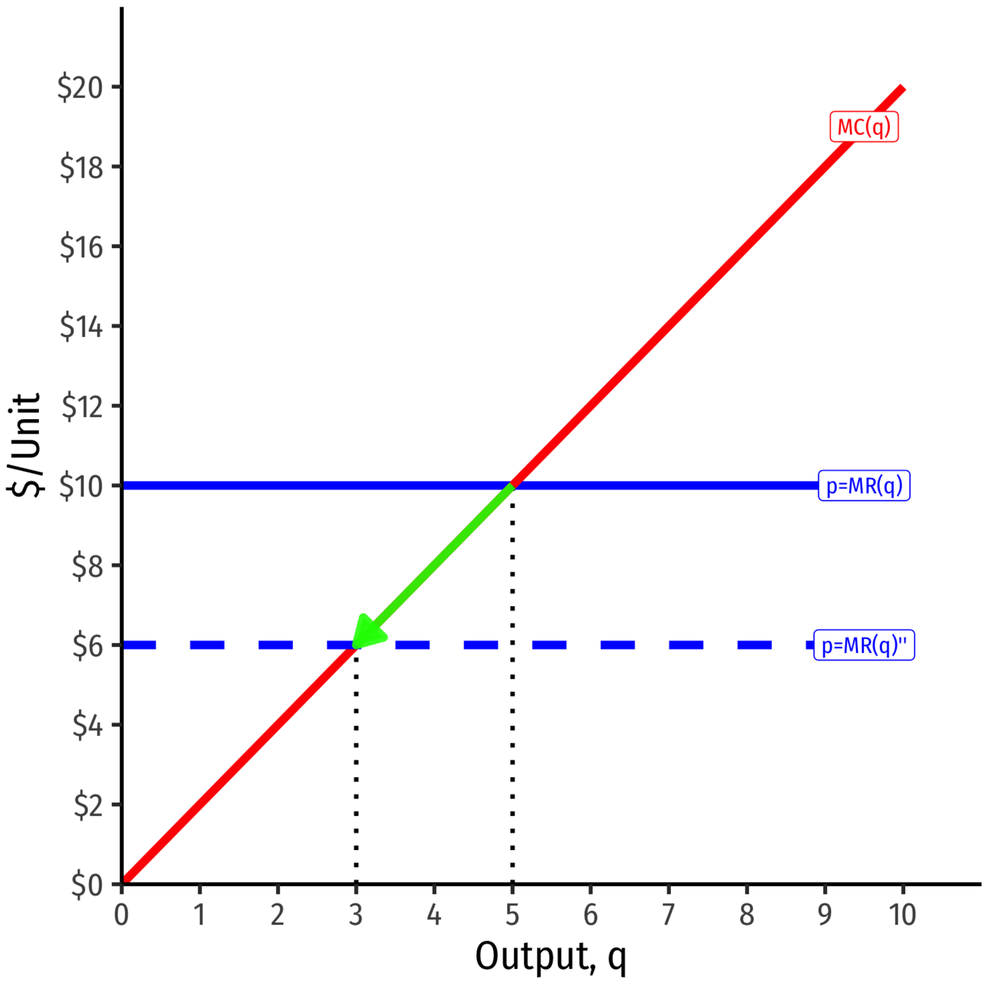
If Market Price Changes I

- Suppose the market price **increases**
- Firm (always setting MR=MC) will respond by **producing more**



If Market Price Changes II

- Suppose the market price **decreases**
- Firm (always setting MR=MC) will respond by **producing less**



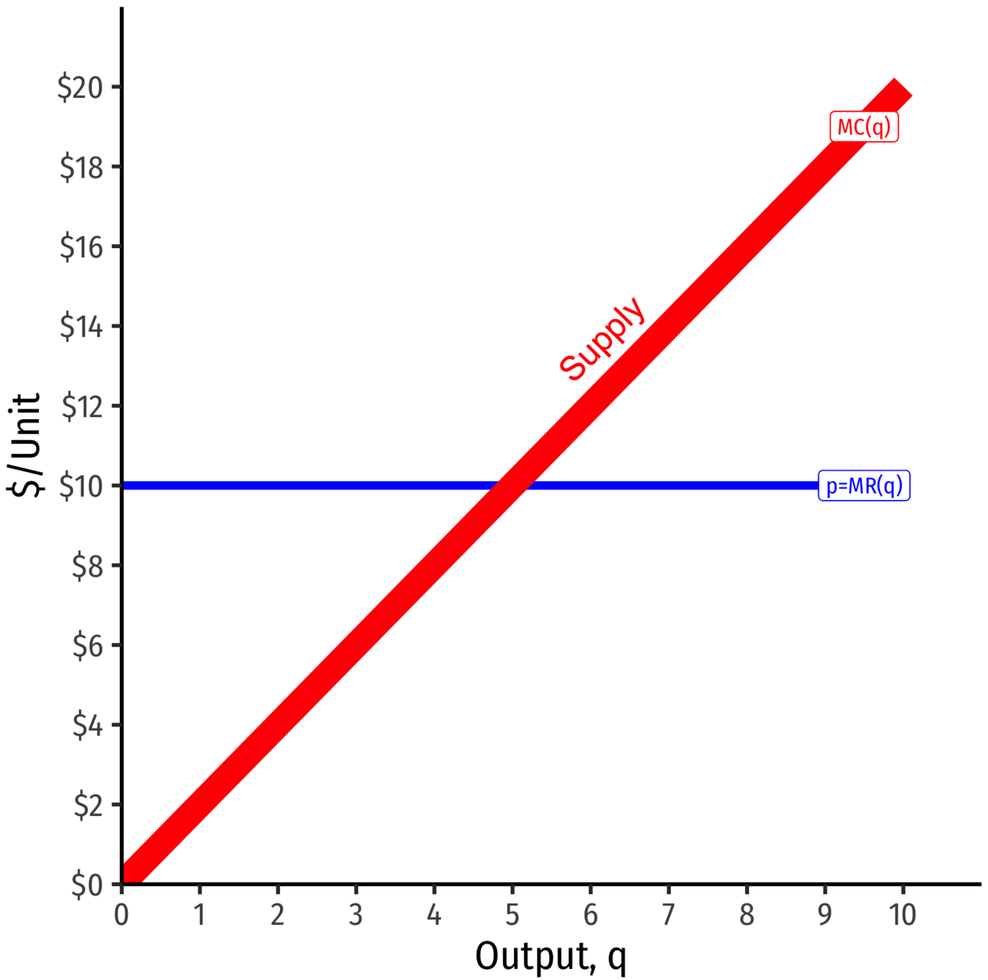
The Firm's Supply Curve

- **The firm's marginal cost curve is its supply curve**‡

$$p = MC(q)$$

  - How it will supply the optimal amount of output in response to the market price
  - **Firm always sets its price equal to its marginal cost**

‡ Mostly...there is an important **exception** we will see shortly!

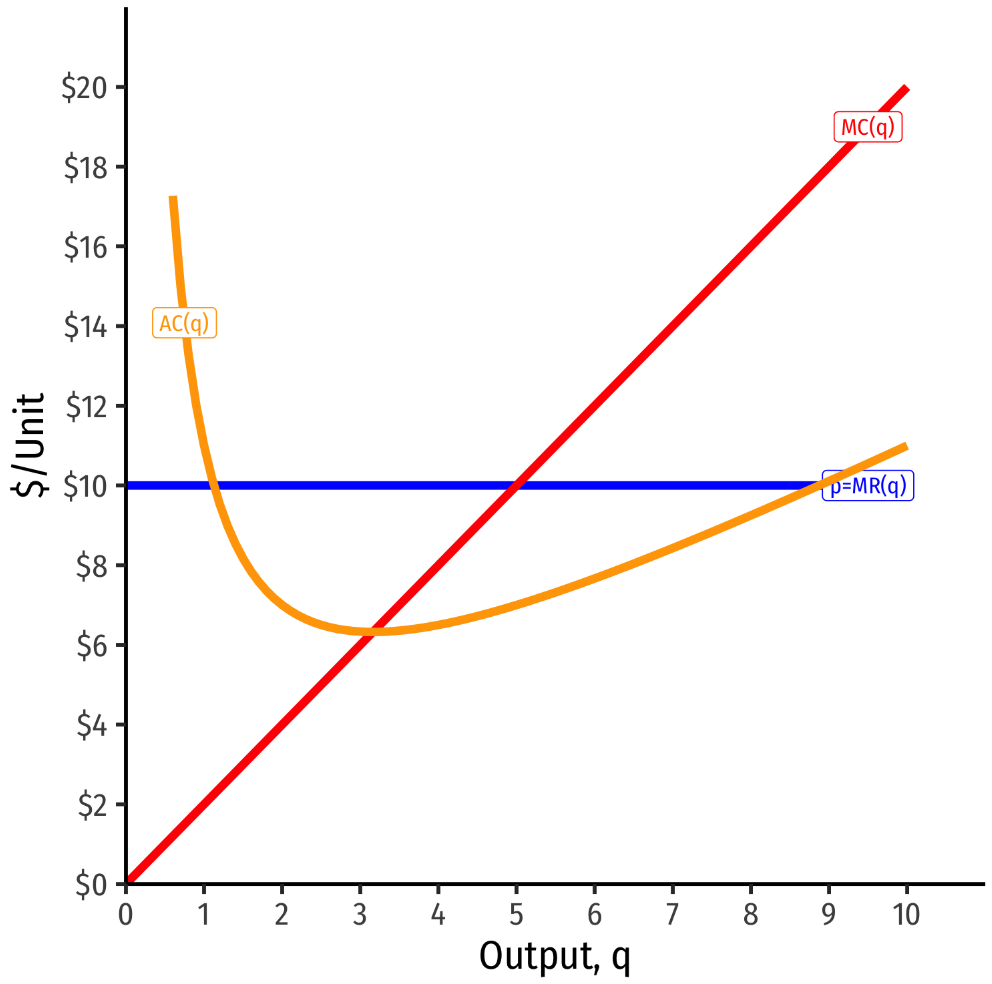


Calculating Profit

Calculating (Average) Profit as AR(q)-AC(q)

- Profit is

$$\pi(q) = R(q) - C(q)$$



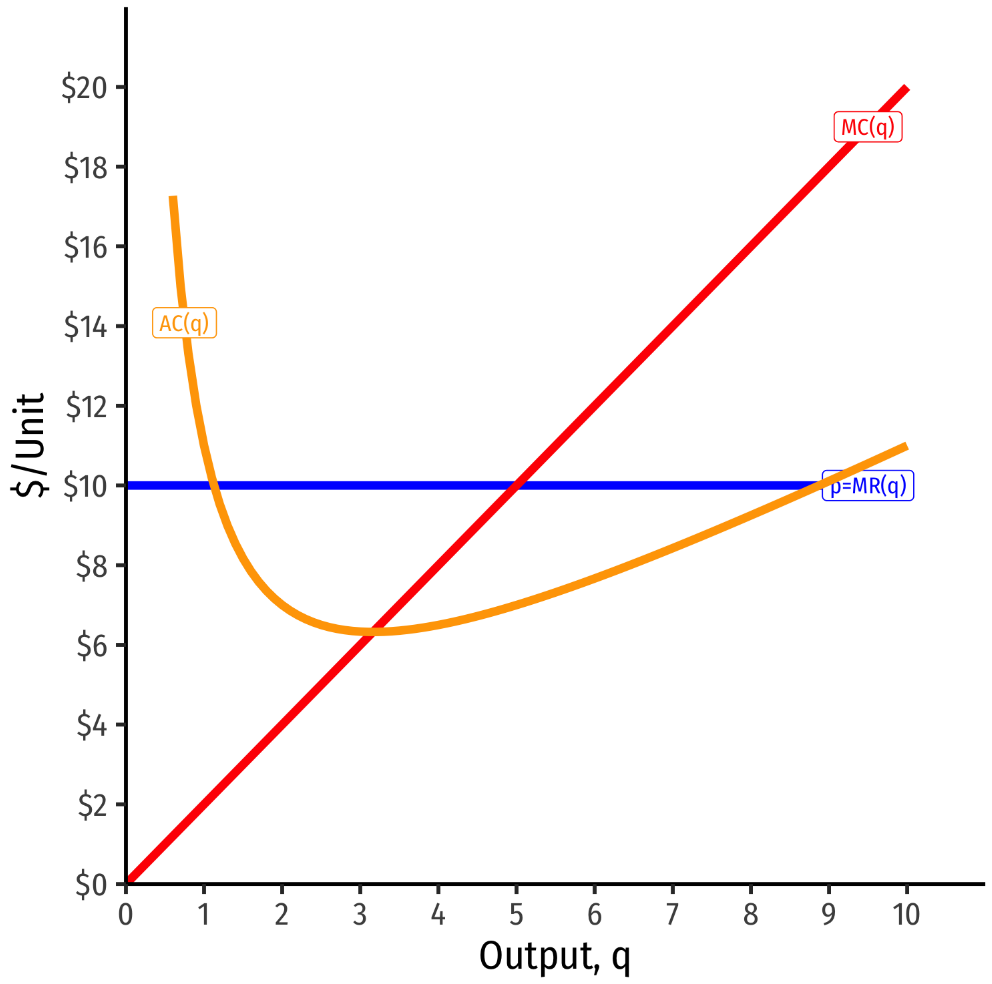
Calculating (Average) Profit as AR(q)-AC(q)

- Profit is

$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\begin{aligned}\frac{\pi(q)}{q} &= {\color{blue}AR(q)} - {\color{orange}AC(q)}\\ &= {\color{blue}p} - {\color{orange}AC(q)}\end{aligned}$$



Calculating (Average) Profit as AR(q)-AC(q)

- Profit is

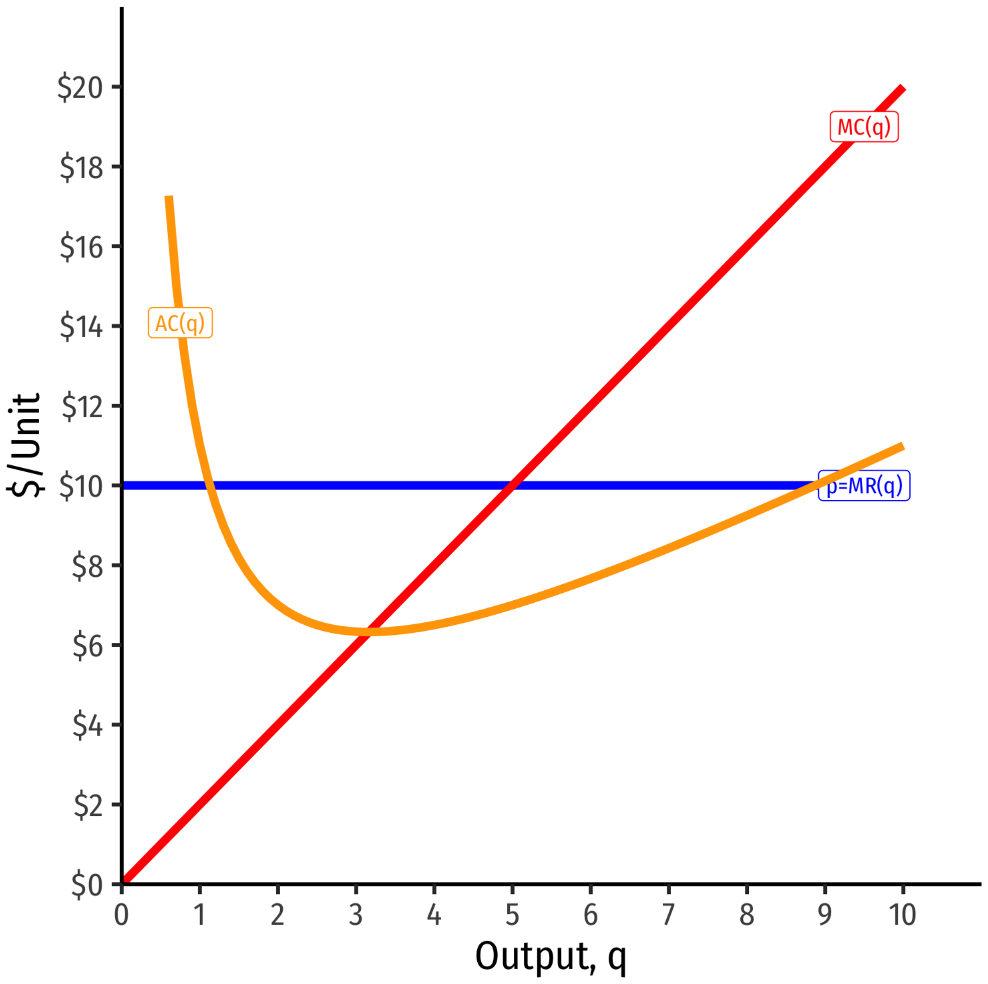
$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\begin{aligned}\frac{\pi(q)}{q} &= {\color{blue}AR(q)} - {\color{orange}AC(q)} \\ &= {\color{blue}p} - {\color{orange}AC(q)}\end{aligned}$$

- Multiply by q to get total profit:

$$\pi(q) = q\left[{\color{blue}p} - {\color{orange}AC(q)}\right]$$

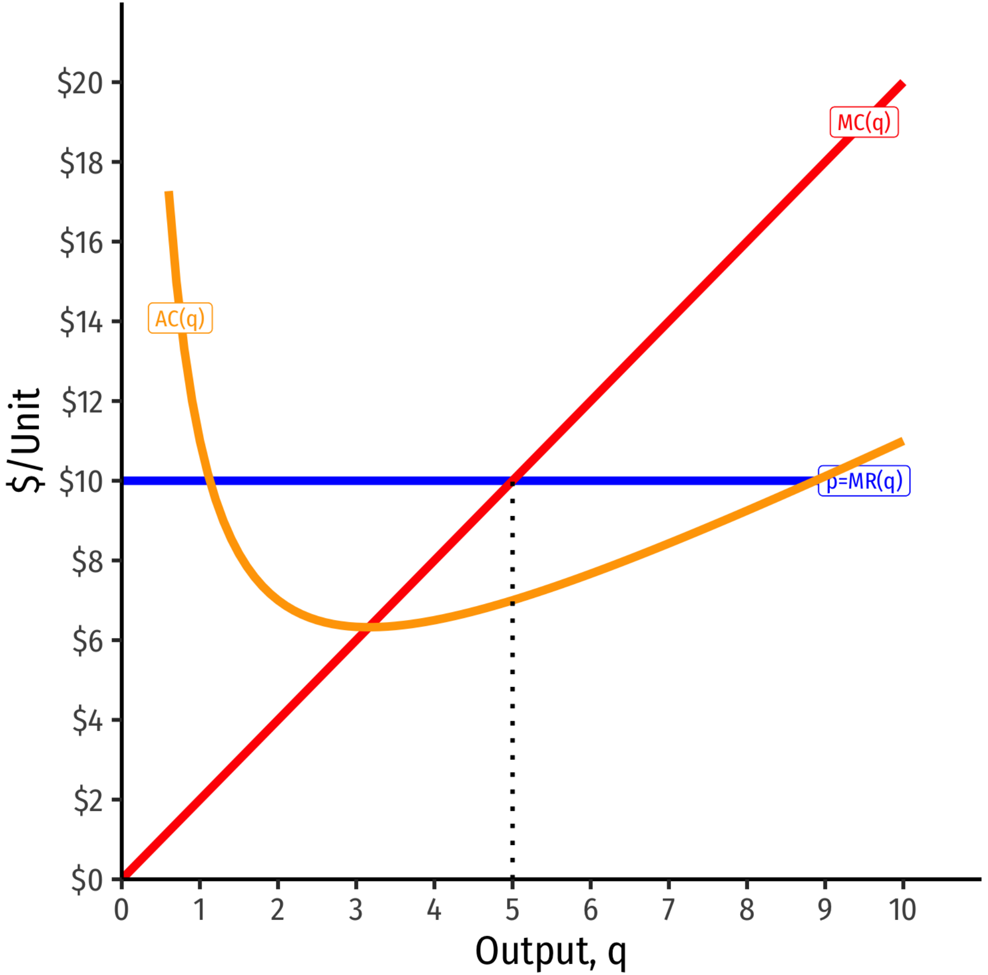


Calculating (Average) Profit as AR(q)-AC(q)

- At market price of $p^* = \$10$

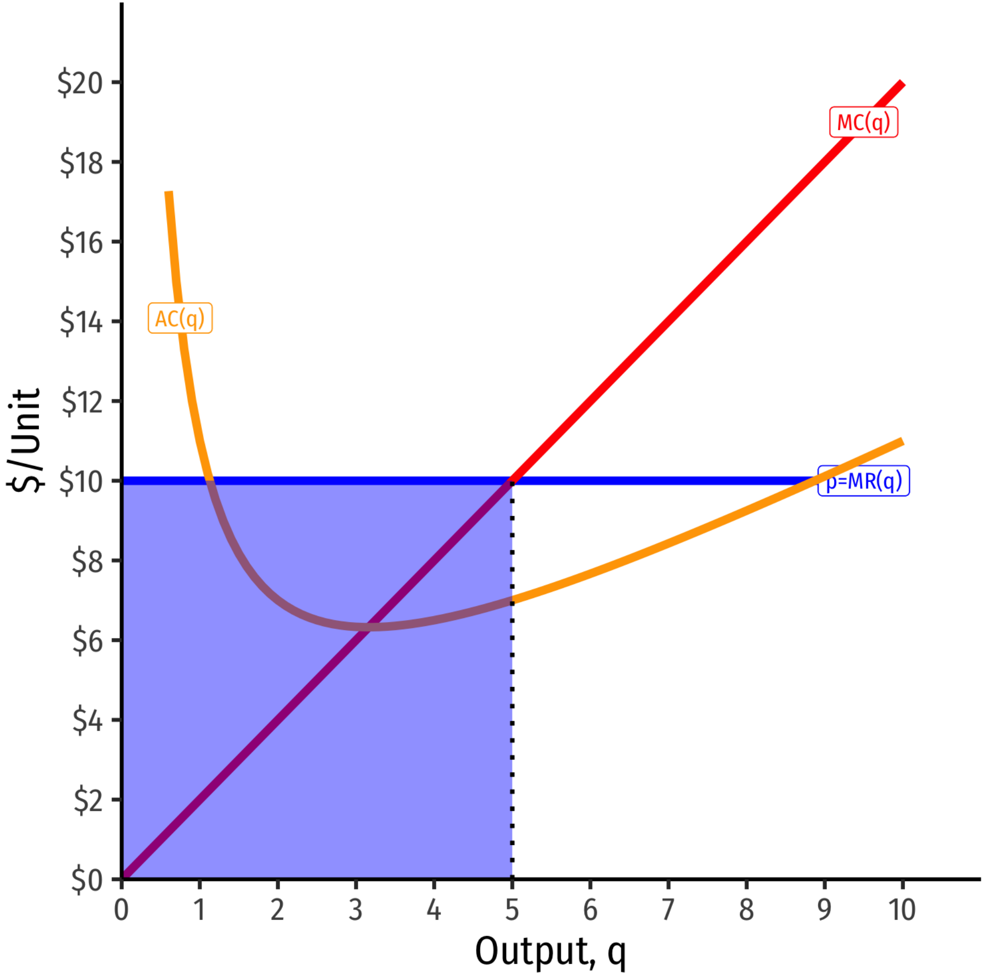
- At $q^* = 5$ (per unit):

- At $q^* = 5$ (totals):



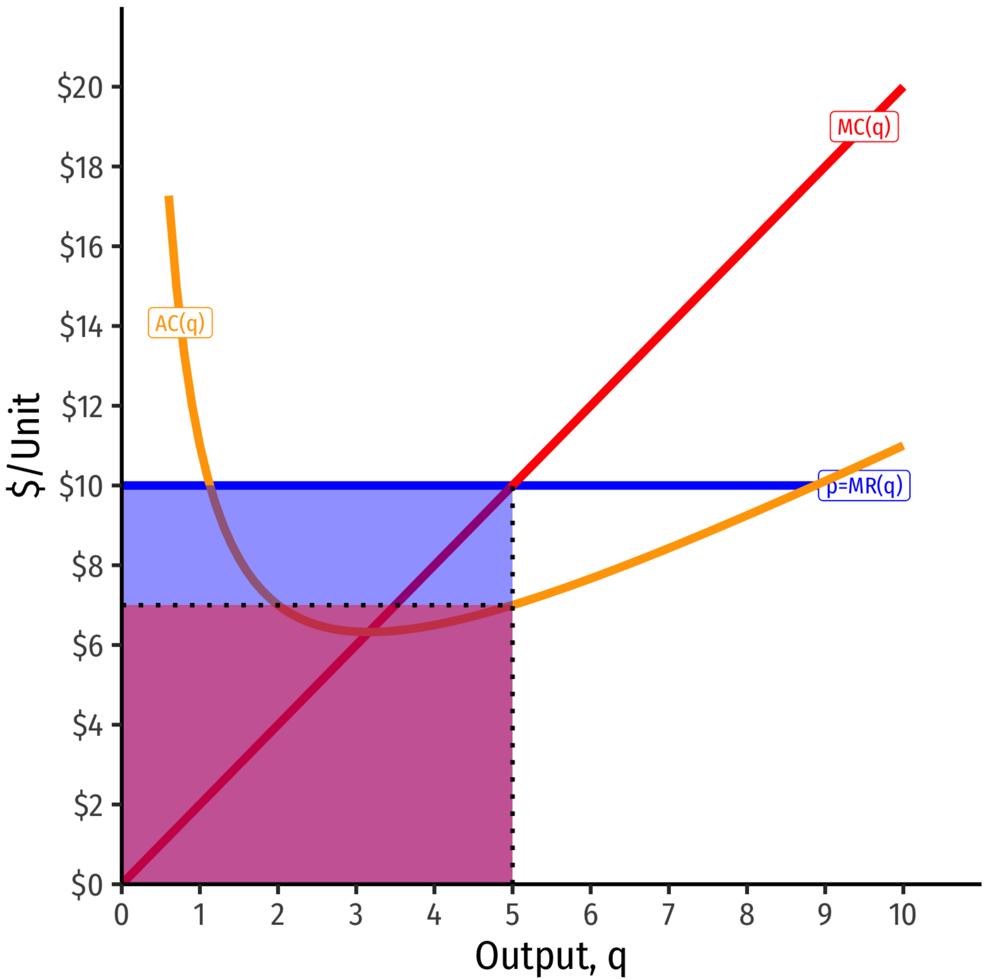
Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $10
- At q* = 5 (per unit):
  - AR(5) = $10/unit
- At q* = 5 (totals):
  - R(5) = $50



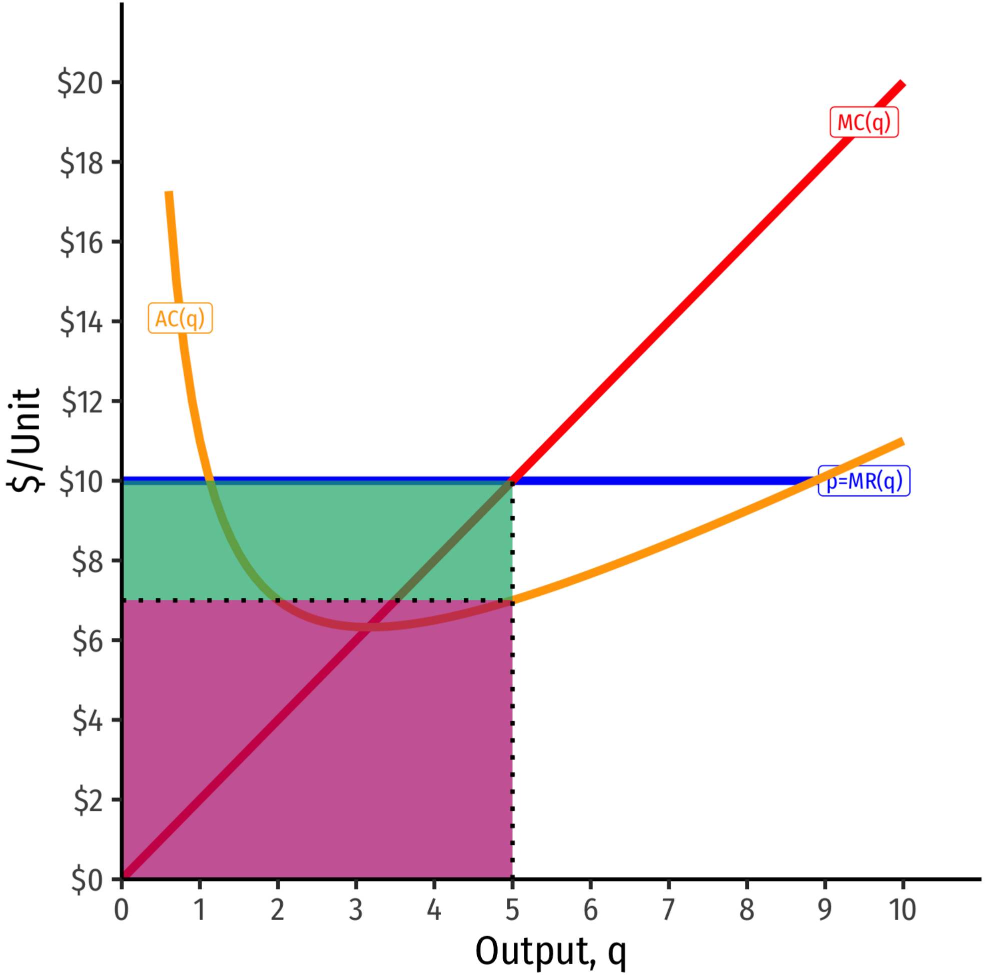
Calculating (Average) Profit as AR(q)-AC(q)

- At market price of $p^* = \$10$
- At $q^* = 5$ (per unit):
    - $AR(5) = \$10/\text{unit}$
    - $AC(5) = \$7/\text{unit}$
- At $q^* = 5$ (totals):
    - $R(5) = \$50$
    - $C(5) = \$35$



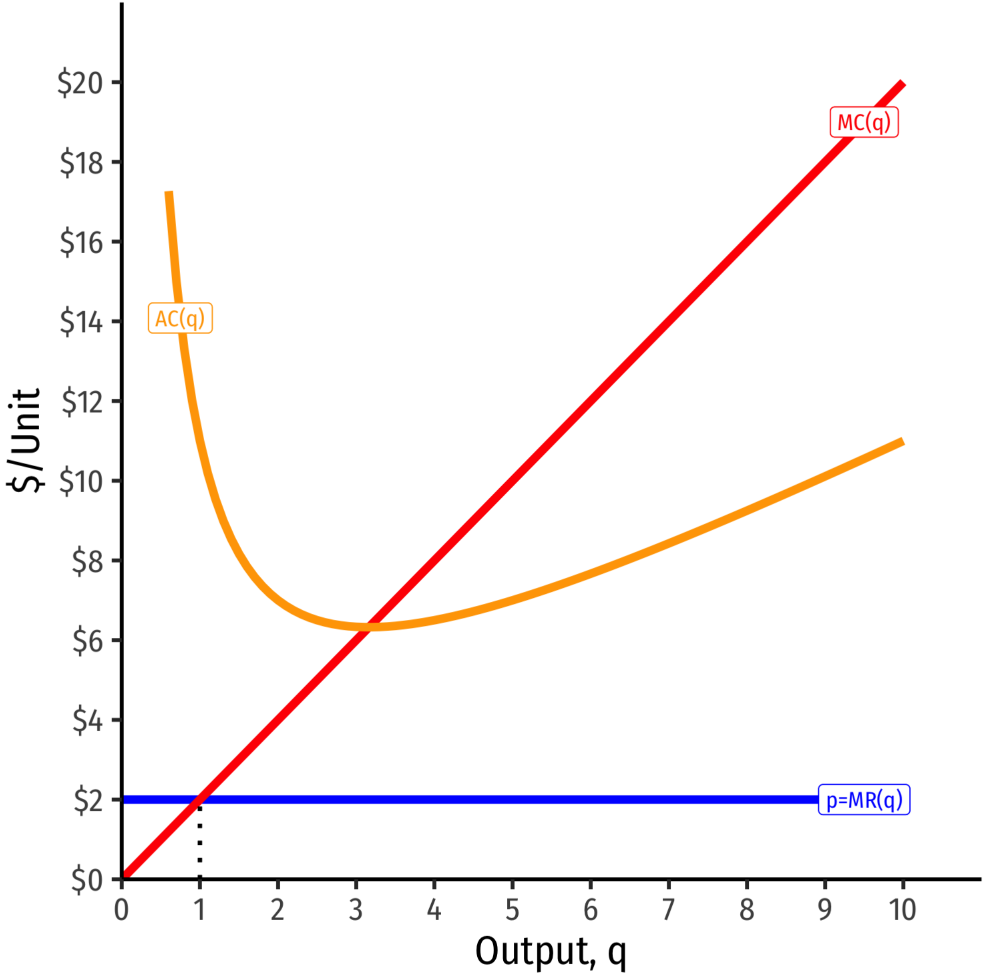
Calculating (Average) Profit as AR(q)-AC(q)

- At market price of $p^* = \$10$
- At $q^* = 5$ (per unit):
  - $AR(5) = \$10/\text{unit}$
  - $AC(5) = \$7/\text{unit}$
  - $A\pi(5) = \$3/\text{unit}$
- At $q^* = 5$ (totals):
  - $R(5) = \$50$
  - $C(5) = \$35$
  - $\pi = \$15$



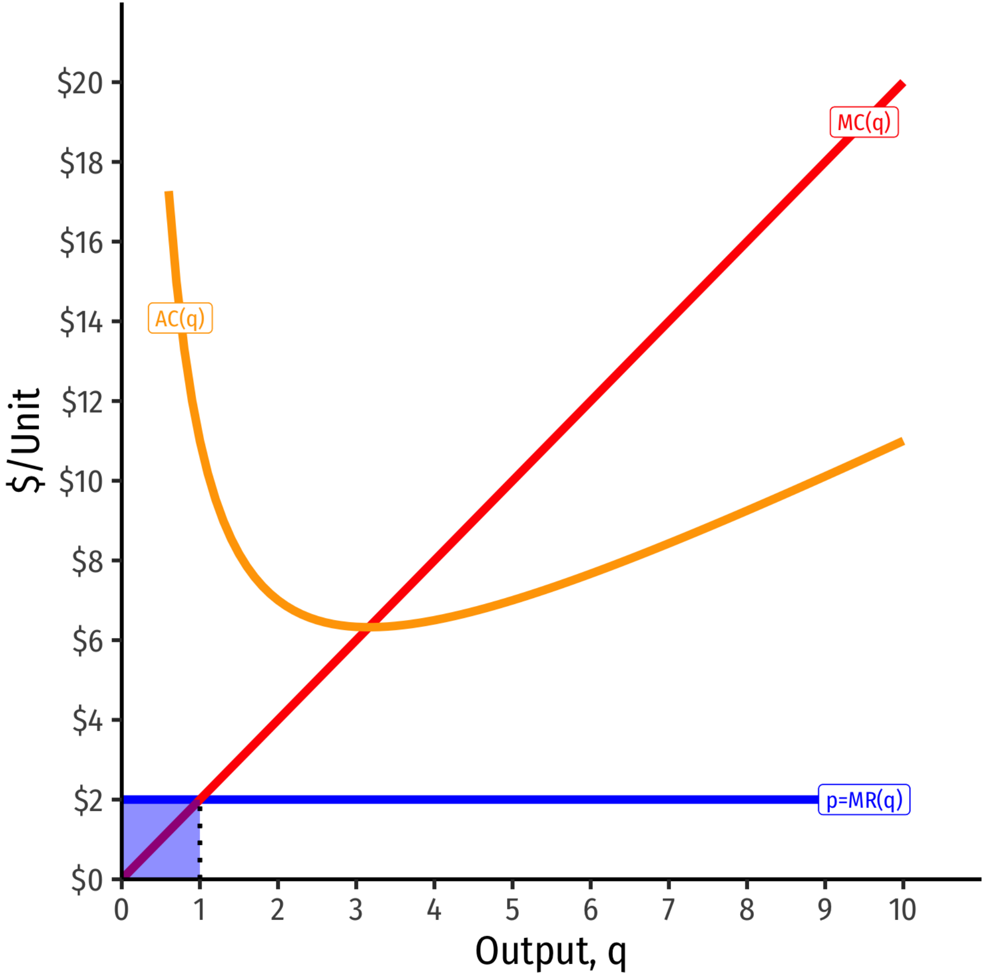
Calculating (Average) Profit as AR(q)-AC(q)

- At market price of $p^* = \$2$
- At $q^* = 1$ (per unit):
- At $q^* = 1$ (totals):



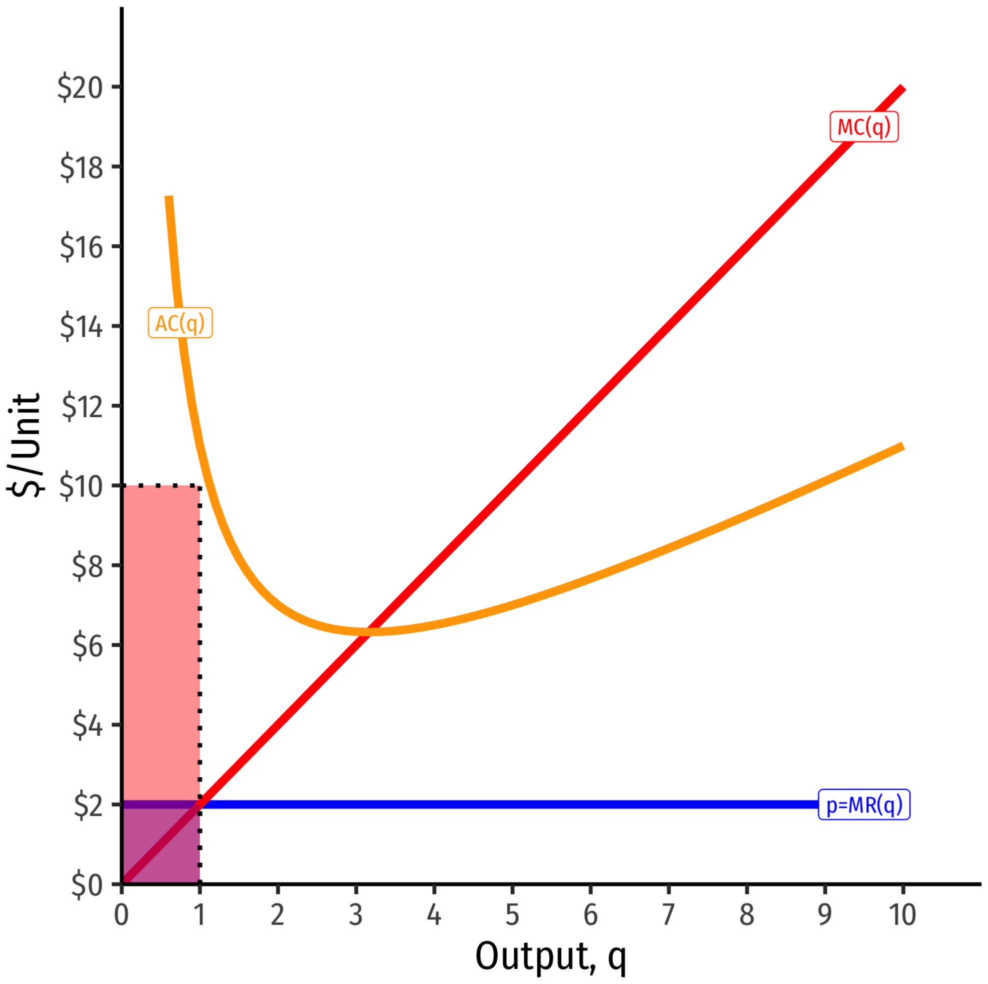
Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $2
- At q* = 1 (per unit):
  - AR(1) = $2/unit
- At q* = 1 (totals):
  - R(1) = $2



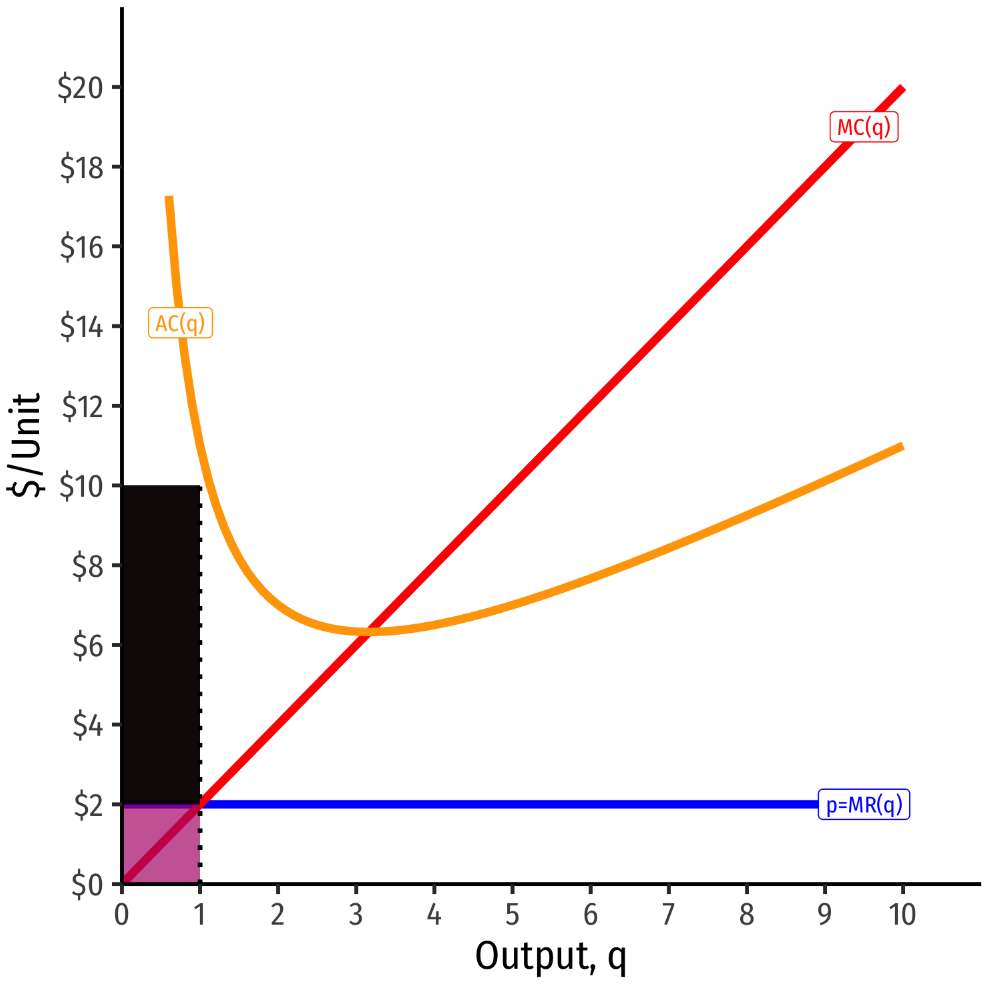
Calculating (Average) Profit as AR(q)-AC(q)

- At market price of p* = $2
- At q* = 1 (per unit):
  - AR(1) = $2/unit
  - AC(1) = $10/unit
- At q* = 1 (totals):
  - R(1) = $2
  - C(1) = $10



Calculating (Average) Profit as AR(q)-AC(q)

- At market price of $p^* = \$2$
- At $q^* = 1$ (per unit):
  - AR(1) = \$2/unit
  - AC(1) = \$10/unit
  - $A\pi(1)$ = -\$8/unit
- At $q^* = 1$ (totals):
  - R(1) = \$2
  - C(1) = \$10
  - $\pi(1)$ = -\$8



Short-Run Shut-Down Decisions

Short-Run Shut-Down Decisions

- What if a firm's profits at q^* are **negative** (i.e. it earns **losses**)?

- **Should it produce at all?**

Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q=0)$:

- If it has **fixed costs** $(f>0)$, its profits are:

$$\pi(q) = pq - C(q)$$

Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q = 0)$:
- If it has **fixed costs** $(f > 0)$, its profits are:

$$\pi(q) = pq - \color{red}{C(q)}$$
$$\pi(q) = pq - \color{red}{f - VC(q)}$$

Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q = 0)$:

- If it has **fixed costs** $(f > 0)$, its profits are:

$$\begin{aligned}\pi(q) &= pq - C(q)\\ \pi(q) &= pq - f - VC(q)\\ \pi(0) &= -f\end{aligned}$$

i.e. it (still) pays its fixed costs

Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q = 0)$ only when:

$$\pi \text{ from producing} < \pi \text{ from not producing}$$

Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q = 0)$ only when:

$$\pi \text{ from producing} < \pi \text{ from not producing}$$
$$\pi(q) < -f$$

Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q = 0)$ only when:

$$\pi \text{ from producing} < \pi \text{ from not producing}$$

$$\pi(q) < -f$$

$$pq - VC(q) - f < -f$$

Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q = 0)$ only when:

$$
\begin{aligned}
\pi \text{ from producing} &< \pi \text{ from not producing} \\
\pi(q) &< -f \\
pq - VC(q) - f &< -f \\
pq - VC(q) &< 0
\end{aligned}
$$

Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q = 0)$ only when:

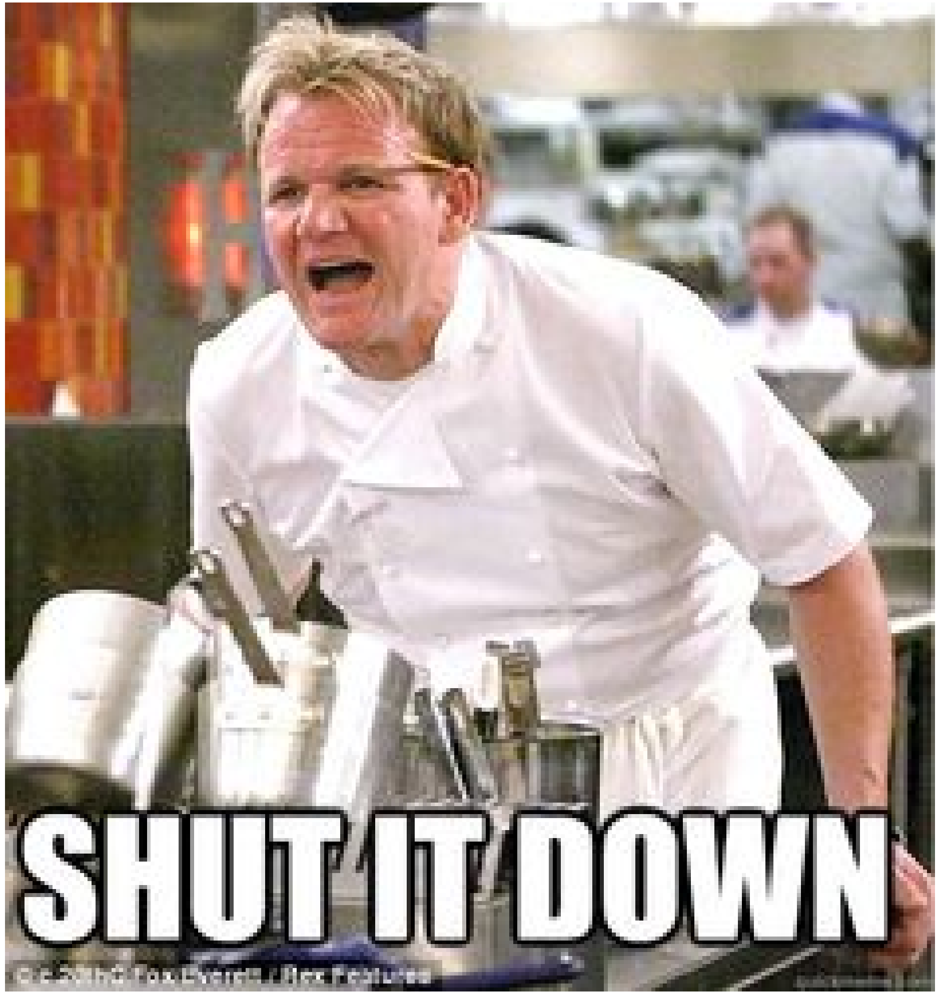
$$\begin{aligned} \pi \text{ from producing} &< \pi \text{ from not producing} \\ \pi(q) &< -f \\ pq - VC(q) - f &< -f \\ pq - VC(q) &< 0 \\ pq &< VC(q) \end{aligned}$$

Short-Run Shut-Down Decisions

- A firm should choose to produce **no output** $(q=0)$ only when:

$$\begin{aligned} \pi \text{ from producing} &< \pi \text{ from not producing} \\ \pi(q) &< -f \\ pq - VC(q) - f &< -f \\ pq - VC(q) &< 0 \\ pq &< VC(q) \\ \color{red}{p} &\color{red}{< AVC(q)} \end{aligned}$$

- **Shut down price**: firm will shut down production *in the short run* when $p < AVC(q)$



The Firm's Short Run Supply Decision

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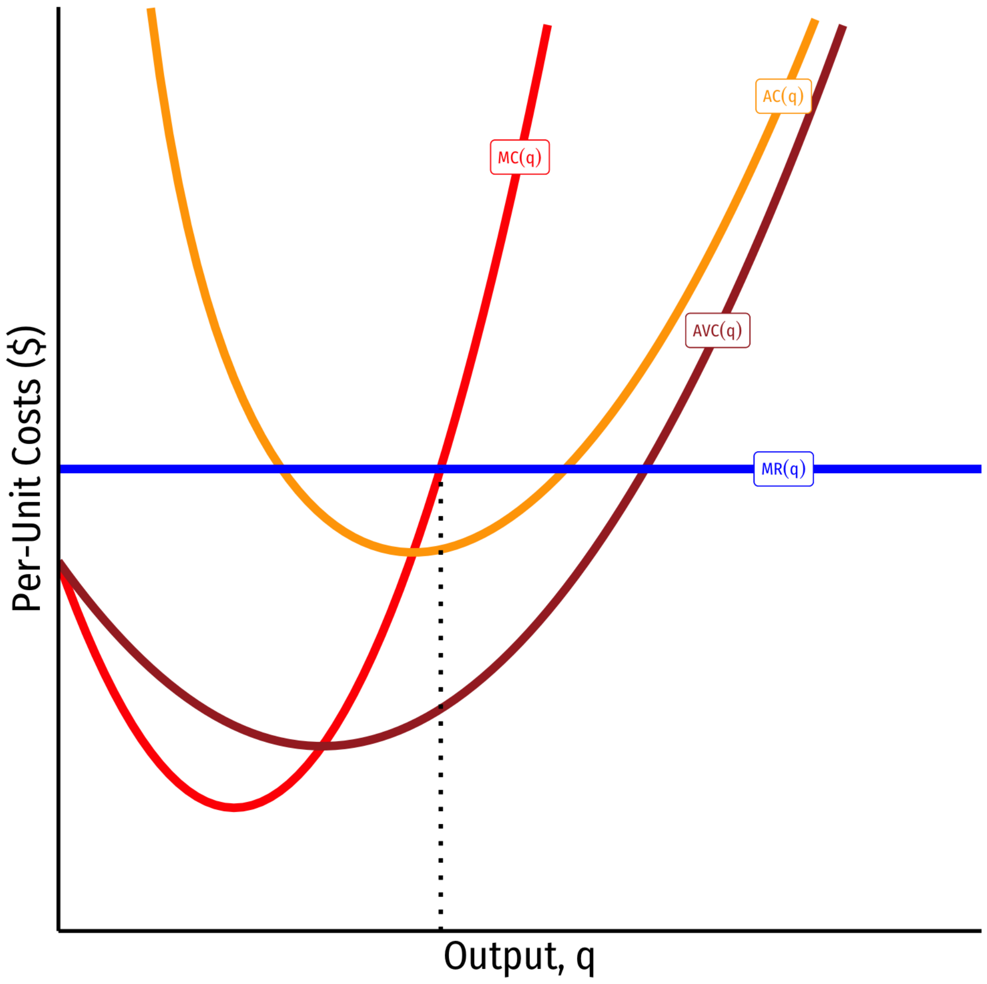
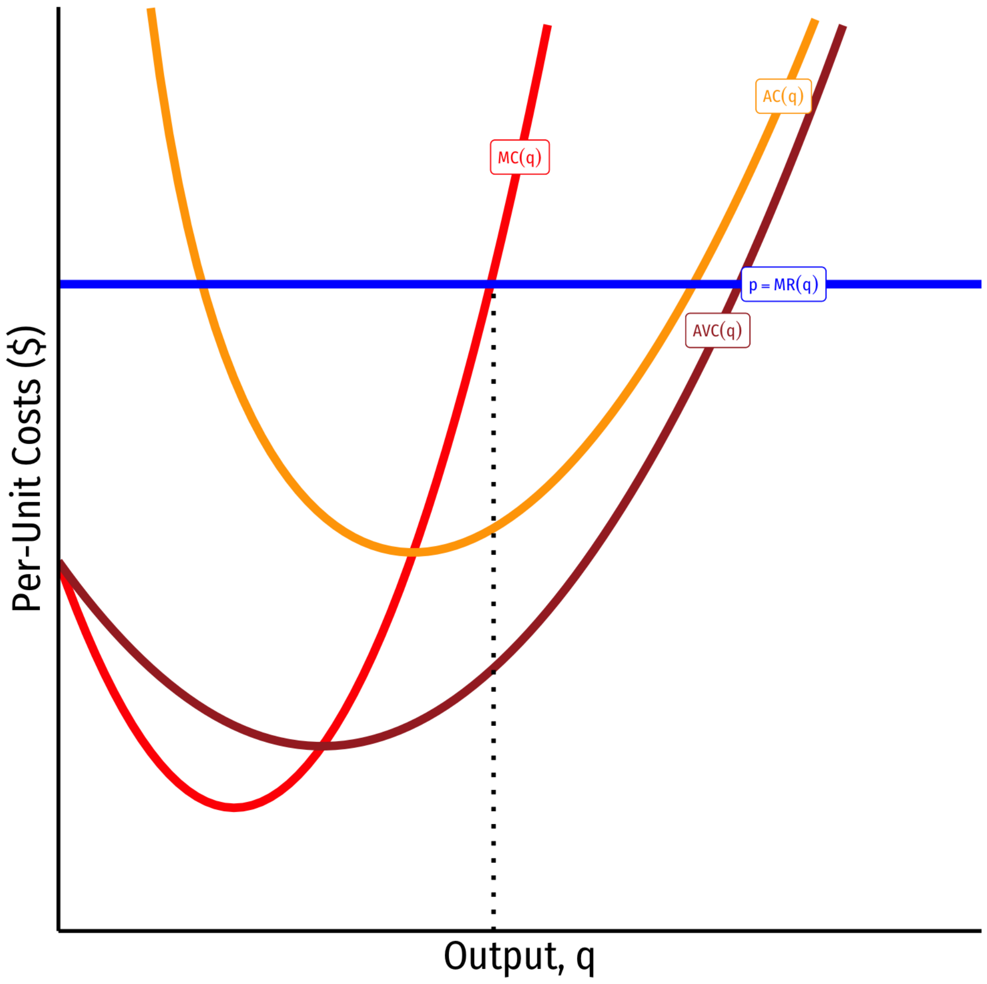


The Firm's Short Run Supply Decision



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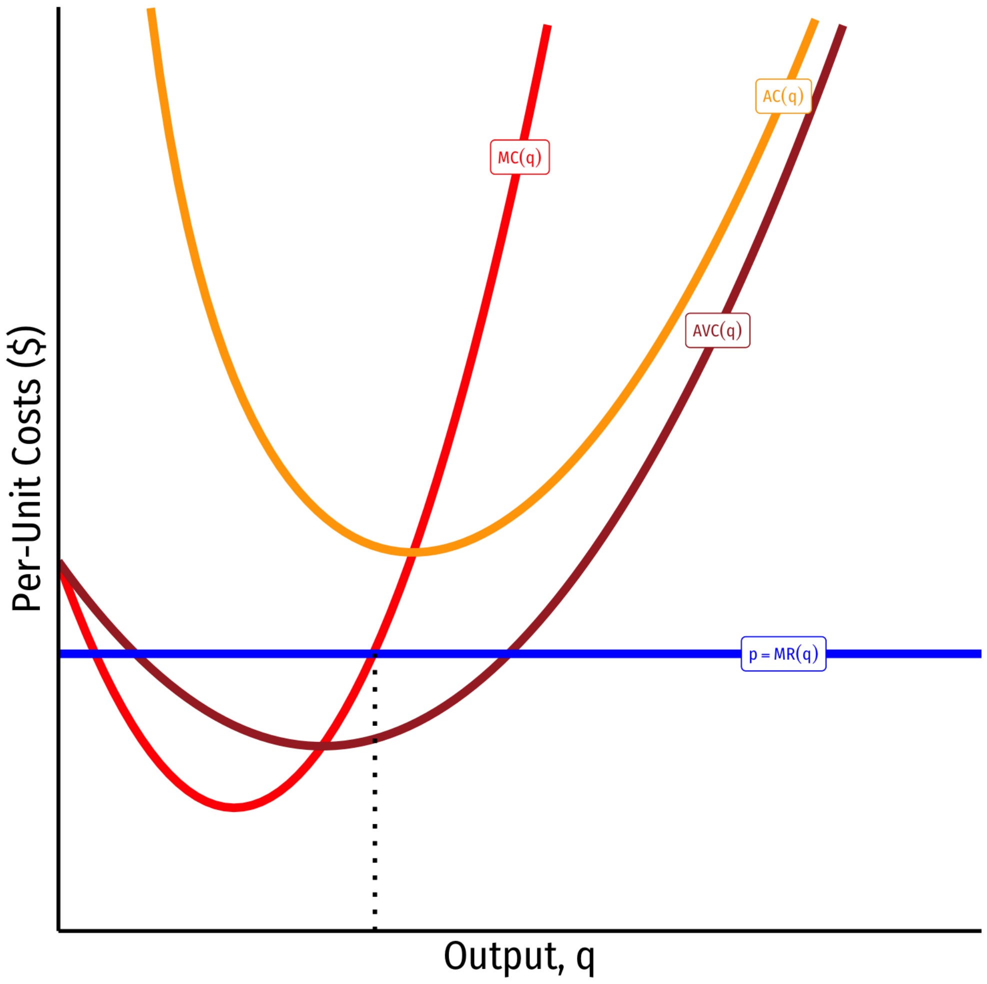
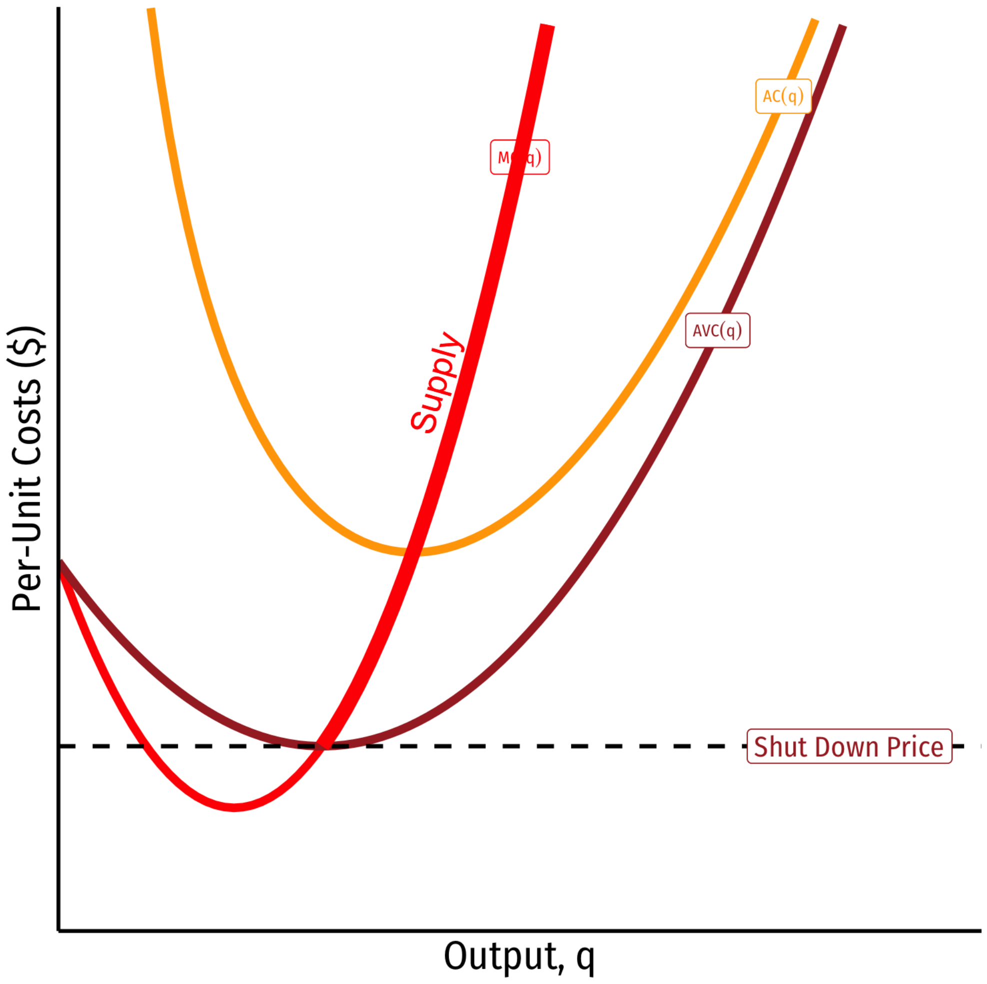
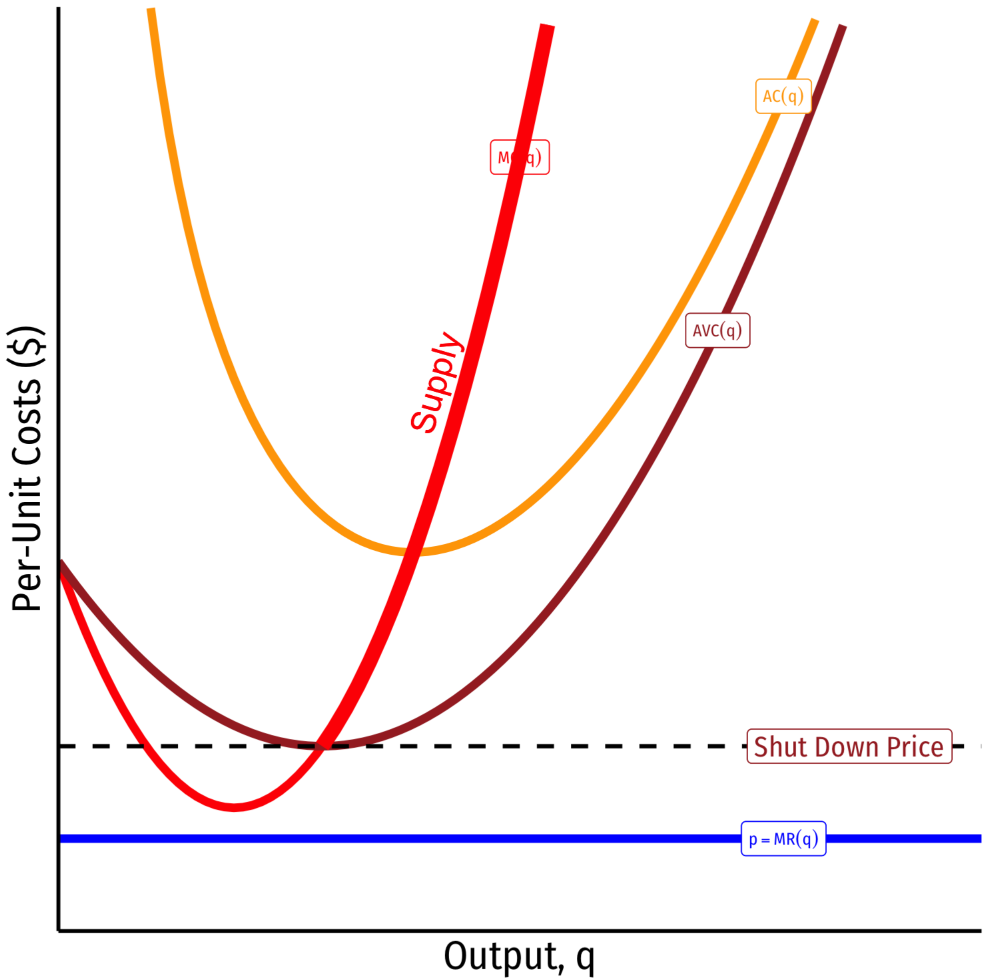


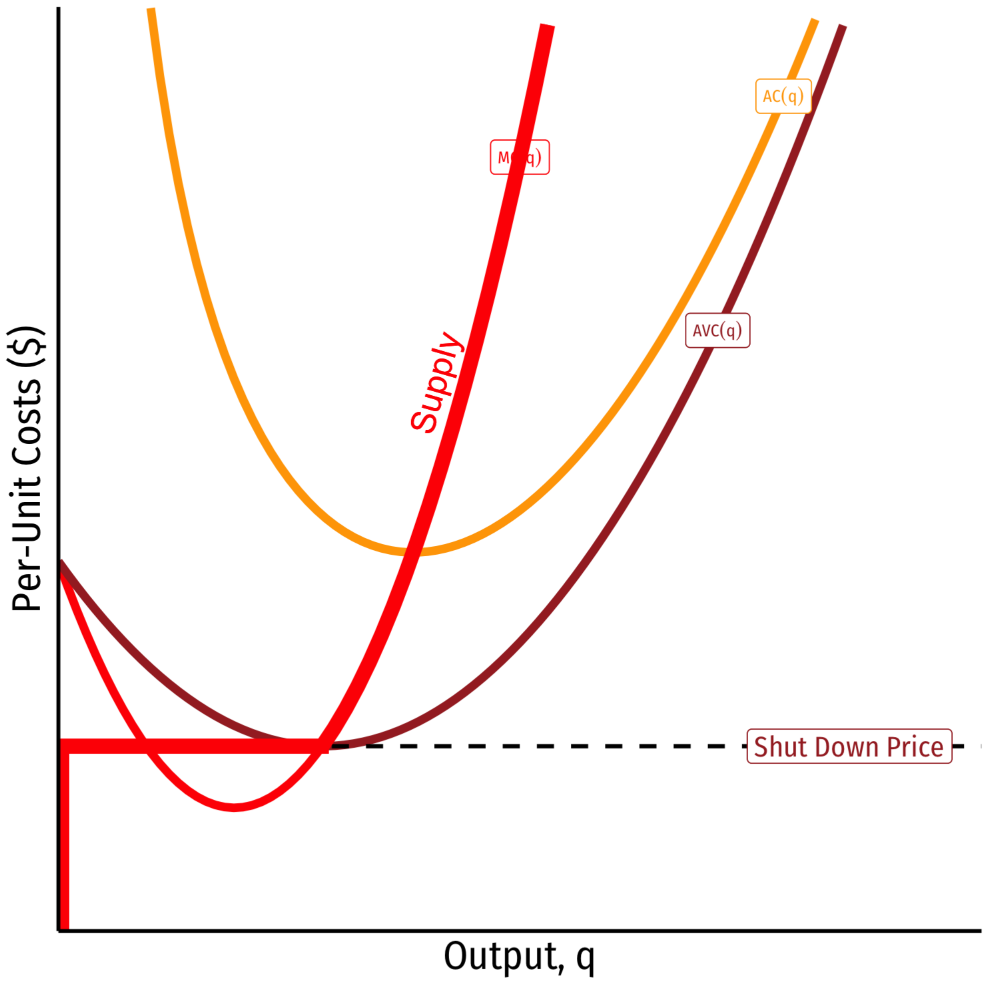
The Firm's Short Run Supply Decision



The Firm's Short Run Supply Decision



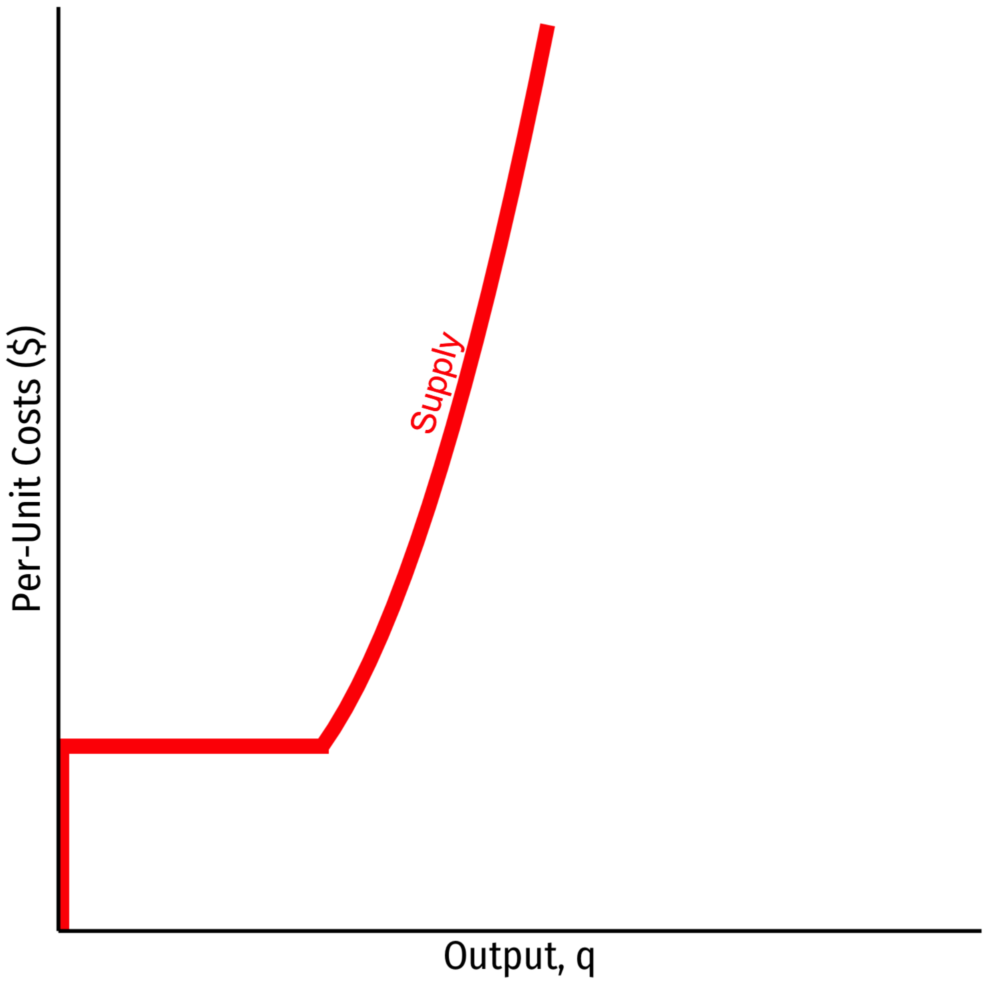
The Firm's Short Run Supply Decision



Firm's short run supply curve:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

The Firm's Short Run Supply Decision



Firm's short run supply curve:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

Summary:

1. **Choose** q^* **such that** $MR(q) = MC(q)$

2. **Profit** $\pi = q[p - AC(q)]$

3. **Shut down if** $p < AVC(q)$

Firm's short run (inverse) supply:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

Choosing the Profit-Maximizing Output q^*: Example

Example: Bob's barbershop gives haircuts in a very competitive market, where barbers cannot differentiate their haircuts. The current market price of a haircut is $15. Bob's daily short run costs are given by:

$$\begin{aligned} C(q) &= 0.5q^2 + 30 \\ MC(q) &= q \end{aligned}$$

1. How many haircuts per day would maximize Bob's profits?

2. How much profit will Bob earn per day?

3. At what price would Bob break even?

4. At what price should the Bob shut down in the short run?

5. Write equations for Bob's short-run supply curve and long-run supply curve